

# STOCHASTIC PROCESSES 26:960:580 Fall 2021 Fridays 10:00-12:50

Michael N. Katehakis <u>Home page</u> 1WP-202 Office Hours: 1 hour before class

## COURSE DESCRIPTION

In this course we study sequences of random variables (stochastic processes) that evolve over time such as a price of a stock, the demand for a commodity, the length of a queue, the credit rating of a corporate bond issuer. For such processes one cannot predict with certainty their future values but we can obtain useful information using a suitable stochastic model to represent how the process evolves over time, and thus obtain the distribution and statistics of future values. The course aims to introduce the fundamentals of stochastic models tools, methods and their applications. The emphasis is on modeling and evaluating uncertainty, simulating complex systems, using data and limited information efficiently, understanding the nature of decision-making, and optimally allocating resources.

## **Course Delivery Mode:**

Most of the classes will be synchronous remote (SR) using <u>Zoom</u>, with portions of prerecorded material.

Learning Management System: Canvas and I will provide additional at: apdalab.org

## Hardware and software requirements:

Notebook pc, basic programing in R or Python.

## COURSE MATERIALS

#### **Required Text:**

S. M. Ross, Applied Probability Models with Optimization Applications. Dover Publications (1992).

#### **Recommended:**

- Sheldon M. Ross: Introduction to Probability Models  $\geq$  7-th Edition, Elsevier
- <u>Essentials of Stochastic Processes</u> by Rick Durrett, Second edition will appear in Summer 2013, published by Springer, available at Rick's page online
- Sheldon M. Ross: Stochastic Processes, J. Wiley, New York, 1995.

#### Other resources:

- ♦ Other Resources: I will provide additional at: reading materials during the course.
- ♦ Start with: Review of Probability.

## PREREQUISITES

- a) Basic undergraduate statistics courses or 22:135:572 Stat for Mgrs.
- b) Calculus with a grade of B or better; or 22:135:571 Calc for Mgrs.

#### LEARNING GOALS AND OBJECTIVES

The objective is to introduce you to the *models and techniques* of Stochastic Processes. This will necessitate some use of theory and computer manipulations of data. However, the goal is not that you will become experts on computer software, but rather to give you a perspective on how stochastic models and techniques are used in practice. Hence, much of the material will be presented in a setting of practical situations. The focus is on ways of modeling and structural ideas about systems that evolve randomly in time.

The course will focus on Chapters 3,4,5,6 and 7 from Ross's text: Poisson processes, renewal processes, discrete and continuous time Markov chains. Additionally, a portion of chapter 9, the Wiener Process and Machine Learning applications will be included. Further, some extra topics such as simulation and applications to optimization of Markov Chains will be introduced. The discussion of Markov chains includes statistical aspects of these processes. The course will begin with a review of basic probability. If time permits, applications to Machine Learning will be introduced.

Specific goals: There will be an introduction to the concepts and applications of those basic ideas that are considered to be most important for the practical analysis of stochastic processes, listed below.

## COURSE SCHEDULE<sup>1</sup>

### Tentative Course Outline.

- $\diamond\,$  Weeks 1-3: Basic concepts
  - (a) Conditional Probabilities and Conditional Expectations.
  - (b) Generating Functions
  - (c) Large Deviations and Insurance problems.
  - (d) The Poisson Process.
    - Exponential Distribution.
    - Defining the Poisson Process.
    - Compound Poisson Processes.
  - (e) Renewal Processes.
    - Laws of Large Numbers.
    - Key Renewal Theorems.
- $\diamond\,$  Week 4: Random Walks
  - (a) Special Examples.
  - (b) Exit Distributions and Exit Times.
  - (c) Harris Paper.

 $<sup>^1\,{\</sup>rm Tentative}.$ 

- $\diamond\,$  Weeks 5-6: Markov Chains.
  - (a) Transient probabilities, Stationary probabilities.
  - (b) Classification of States.
  - (c) Drifts and Limit Behavior.
  - (d) Unstable Systems.
  - (e) Geometric Ergodicity Papers.
  - (f) Semi-Markov Chains.
- $\diamond\,$  Weeks 9-10: Markov Decision Processes.
  - (a) Examples, Basic Properties.
  - (b) Optimality Criteria.
  - (c) Value functions and Solution Methods.
  - (d) Inventory Control Models.
  - (e) Random Supply Models.
- ♦ Weeks 11-13: Applications Martingales and Finance.
  - (a) Gambling Strategies, Stopping Times.
  - (b) The Multi Armed Bandit Problem.
  - (c) Martingales Examples, Basic Properties.
  - (d) Binomial Model.
  - (e) Capital Asset Pricing Model.
  - (f) American Options
  - (g) Black-Scholes formula
  - (h) Calls and Puts
  - (i) Infinitesimal look ahead rules.

## ACADEMIC INTEGRITY

Students are responsible for understanding the <u>RU Academic Integrity Policy</u>. Students must sign the RU Honor Pledge. See business.rutgers.edu/ai for more details.

#### ATTENDANCE AND PREPARATION Adjusted for Current Conditions

Grade performance is a demonstrated function of attendance, preparation and participation. You can get behind very easily by skipping classes, resulting in a poor understanding of the material, which will show up as a poor grade for the class. Any class sessions missed by the student are the student's responsibility to make up, not the instructor's. Late arrival that causes disruption, early departure that causes disruption, excessive conversation among students (a disruption in its own right), inappropriate use of electronic devices that cause disruptions, and other actions that disrupt the classroom are unacceptable.

If I am to be absent, my department vice chair or I will send you notice via email and Canvas as far in advance as possible. If you are to be absent, report your absence in advance at https://sims.rutgers.edu/ssra/. If your absence is due to religious observance, a Rutgers-approved activity, illness, or family emergency/death and you seek makeup work, also send me an email with full details and supporting documentation within 3 days of your first absence.

Expect me to arrive on time for each class session. I expect the same of you. Expect me to remain for the entirety of each class session. I expect the same of you.

## EXAMS AND GRADING

Your grade<sup>2</sup> will be based on a final exam, a quiz, case write-ups, homework assignments, and class participation, as follows:

- $\diamond$  Mid Term exam (3-hour exam) 40%
- $\diamond$  Final exam (3-hour exam) 50%
- $\diamond$  Homework and class participation and conduct 10%

All exams will be take-home.

#### 'CLASSROOM' CONDUCT

The course will be largely taught using <u>Zoom</u> presentation. Class-related material (lecture notes, messages, etc.) will be posted on the class site. Additional visual material and demos may be shown in some classes. Most of your work will take place outside the classroom, as you study, and apply the material to which you are introduced in class.

**Homework Assignments** are designed to help you learn the material discussed in class. In addition doing a thorough job on the homework assignments is the best preparation for the quiz and the final examination. There are three types of assignments: read, prepare, and hand in.

- **Read:** When the assignment is to read some material, this reading is an important introduction to the topics to be discussed in class. I will make the assumption that you have done the reading before class and have understood much (but not necessarily all) of it. When the assignment is to read a problem, that problem will often be used in class to introduce new concepts.
- **Prepare:** Fully analyze the problem. Be ready to discuss it in class using <u>Zoom Share-Screen</u> share screen, with the numbers computed, etc. I will call on people, so please be ready.
- Hand In: The same as prepare, but you must turn in your analysis. All written assignments must be handed in at the beginning of class on the day they are due, and so you will probably want to make a copy of your assignment for reference during class. All written assignments will be graded. These assignments should be submitted in typed form using a word processor. Please write your name, RUID and email on all homework submitted. Team work on this homework is not allowed. Unless a documented reason is produced for unusual circumstances, late submissions will not be accepted more than a week late.

Your class participation will be evaluated subjectively, but will rely upon measures of punctuality, attendance, familiarity with the required readings, relevance and insight reflected in classroom questions, and commentary. Your class participation will be judged by what you add to the class environment, regardless of your technical background. Although several lectures will be didactic, we will rely heavily upon interactive discussion within the class. Students will be expected to be familiar with the readings, even though they might not understand all of the material in advance. In general, questions and comments are encouraged. Comments should be limited to the important aspects of earlier points made, and reflect knowledge of the readings. You may called on to answer questions about the homework or classroom discussion. Your classroom participation evaluation is based on the extent to which you contribute to the learning environment. However, correcting a mistake of the professor and asking what appear to be "dumb questions" about what is being covered are positive contributions. In the case

<sup>&</sup>lt;sup>2</sup>Your final grade is not subject to negotiation. If you feel I have made an error, submit your written argument to me within one week of receiving your final grade. Clarify the precise error I made and provide all due supporting documentation. If I have made an error, I will gladly correct it. But I will adjust grades only if I have made an error.

of so-called "dumb questions," very often half of the class will have the same questions in mind and are relieved to have them asked.

**Other requirements are:** On-time arrival to the <u>Zoom</u> classes, with uninterrupted attendance for the duration. Maintenance of a professional atmosphere - use respectful comments and *mute the microphone when not speaking*. Refraining from distracting or disrespectful activities, e.g., avoiding side conversations. Courtesy towards all participants in the **Zoom** 'classroom'.

If your absence is due to religious observance, a Rutgers-approved activity, illness, or family emergency/death and you seek makeup work, also send an email with full details and supporting documentation within 3 days of your absence.

#### SUPPORT SERVICES

If you need accommodation for a disability, obtain a Letter of Accommodation from the Office of Disability Services. The Office of Disability Services at Rutgers, The State University of New Jersey, provides student-centered and student-inclusive programming in compliance with the Americans with Disabilities Act of 1990, the Americans with Disabilities Act Amendments of 2008, Section 504 of the Rehabilitation Act of 1973, Section 508 of the Rehabilitation Act of 1998, and the New Jersey Law Against Discrimination. https://ods.rutgers.edu

If you are a military veteran or are on active military duty, you can obtain support through the Office of Veteran and Military Programs and Services. http://veterans.rutgers.edu/

If you are in need of mental health services, please use our readily available services. [Select for inclusion in syllabus based on course location] [Rutgers University-Newark Counseling Center: http://counseling.newark.rutgers.edu/] [Rutgers Counseling and Psychological Services ? New Brunswick: http://rhscaps.rutgers.edu/]

If you are in need of physical health services, please use our readily available services. [Select for inclusion in syllabus based on course location] [Rutgers Health Services ? Newark: http://health.newark.rutgers.edu/] [Rutgers Health Services ? New Brunswick: http://health.rutgers.edu/]

If you are in need of legal services, please use our readily available services: http://rusls.rutgers.edu/