Pricing Residential Mortgage Credit Risk in the Post-GFC Era*

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Abstract

Since the collapse of the subprime residential mortgage market in the GFC, the U.S. mortgage market has seen the rebirth of a new residential mortgage risk asset class: the Credit Risk Transfer bond market. We provide a new asset pricing framework for pricing CRT bonds in a manner consistent with the pricing of risk in the treasury bond, corporate bond, and housing markets. We find that the GSEs have paid the CRT bond investors fairly on average, too much for the low-risk tranches and not enough for the highest-risk tranches. We use our framework to assess fair compensation for credit risk on the overall U.S. residential mortgage pool and within the cross-section of households. We find that the large increases in average g-fees since the GFC have brought the g-fees in line with the underlying credit risk. In the cross-section of borrowers, high-credit risk borrowers with low FICO scores cross-subsidized low-risk borrowers. While a 2023 reform partially corrected the cross-subsidization in the FICO dimension, the reform introduced cross-subsidization from low-LTV to high-LTV households.

Keywords: mortgage default risk, prepayment risk, GSE reform, cross-subsidization in household finance

JEL: G12, G21, R31

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1 Introduction

One of the largest financial catastrophes of the last fifty years was the implosion of the private-label U.S. residential mortgage-backed securities market. The government bailed out the banks that had exposure to risky mortgage credit and took the mortgage giants Fannie Mae and Freddie Mac (the GSEs), who had been swept up in the subprime credit bonanza, into conservatorship (Acharya et al., 2011). Since then, the majority of U.S. mortgage credit risk has been born by the taxpayers.[1]

Starting in 2013 and largely out of the public's eye, the GSEs began to lay off the credit risk on the mortgages they had acquired to the private sector. They did so in the form of credit risk transfer (CRT) bonds. What started as a pilot program in 2013 grew into a new asset class, as it came to cover 90% of all newly originated conforming mortgage debt. Since 2013, the GSEs have issued \$145 billion of CRT bond principal, passing back most of the credit risk on \$5.2 trillion of mortgage principal back to the private sector.

What that means is that the U.S. mortgage market has, to a large degree, been *functionally re-privatized*. The GSEs purchase mortgages through the front door, synthetically split the mortgage into two components, one covering the interest rate and prepayment risk and a second component covering the default risk, and sell both pieces off through the back door in the form of default-free agency mortgage-backed securities and credit risk transfer bonds, respectively. If we think of the GSEs prior to 2013 as a large insurer of mortgage credit risk, we can think of the GSEs post-2013 as re-insuring a large share of the credit risk with the private sector. The focus of this paper is on the CRT market where most of that re-insurance takes place. Since the demise of the subprime mortgage market during the GFC, it has become the only market where residential mortgage credit risk is traded in significant quantity. Despite its considerable size and importance as a barometer of residential credit risk, this asset class is understudied.

We develop a state-of-the-art pricing model for CRT bonds that takes into account the myriad institutional features of these bonds. Our asset pricing model is consistent with the prices of short-term and long-term government bonds and investment-grade corporate bonds. That is, we assume that the marginal agent in the CRT bond space is an investor who also invests in U.S. government and corporate debt, consistent with anecdotal evidence.[2] In the language of asset pricing, we extract the market prices of interest rate and credit risk from the Treasury and corporate debt markets. Our model also stipulates the dynamics of aggregate labor income risk, layoffs, and aggregate house price growth. We insist on matching the risk premium in the housing market, a key parameter.

We model mortgage default as triggered by the combination of a liquidity (disposable income)

[1] The share of new mortgage originations held on banks' balance sheets grew from below 15% in 2009 to 30% in 2024. Most of this privately-held mortgage debt consists of jumbo mortgages with loan balances above the conforming loan limit and very low default risk. The share of originations of mortgages guaranteed by the Federal Housing Administration and the Veteran's Administration jumped to 20% in the post-GFC period. The share of GSE debt was around 60% between 2008 and 2013, around 45% between 2014 and 2019, 60% in 2020-2021, and around 40% in 2023-24.

[2] Indeed, industry publications from investment banks usually discuss CRT bond spreads relative to corporate credit spreads, since investors in CRT bonds tend to invest in corporate debt as well.

shock and negative home equity. Households face both idiosyncratic and aggregate income and house price risk. The default model produces realistic average mortgage default rates for the various GSE mortgage vintages, and across credit score (FICO) and loan-to-value (LTV) groups. Since mortgage prepayments extinguish the mortgage, they also extinguish the associated credit risk. Hence, we build a prepayment model that fits the patterns of empirical prepayment across vintages and coupons well. Prepayments affect the cash-flows to the various CRT bonds in non-trivial ways and are a key component of the valuation exercise.

With the model in hand, our main exercise is to price each of the 329 CRT bonds issued between 2013 and 2025 by Freddie Mac. These bonds differ by vintage, underlying mortgage collateral, and seniority (usually 3–5 tranches per deal). We use loan-level microdata to infer the initial composition of each CRT mortgage pool.

Our first main result is that, on average across the 329 bonds, our model implies credit spreads that closely match the spreads investors received in the data. The average gap between data and model across the 329 bonds is a mere 3.1 basis points. We obtain this fair pricing result despite the fact that the asset pricing model did not explicitly target CRT or any other mortgage bond prices. Contrary to received wisdom, the government has not offered excessively high compensation for transferring credit risk to the private sector.

We arrive at this conclusion because the model generates substantial housing and credit risk premia. These risk premia result in large wedges between mean aggregate income and house price growth rates under the risk neutral and physical measures. Since the pricing of mortgage default risk depends critically on the evolution of income and house prices under the risk-neutral measure, our model delivers high credit spreads for the CRT tranches. While *realized* default rates may have been fortuitously low over the past 12 years, investors were appropriately compensated for the *ex-ante* risk of high default rates.

Our second main result is that while the average CRT bond was priced fairly, that average hides interesting variation between junior and mezzanine tranches. The junior tranches, which take losses first, have model-implied spreads that are about 340 basis points (bps) higher than in the data, whereas the mezzanine tranches, which take losses only after the junior tranches are wiped out, have model-implied spreads that are 150 bps lower than in the data. Put differently, CRT investors in the junior bonds have received spreads that are too low; these bonds were overpriced. Mezzanine tranches were underpriced, offering attractive yields to investors given the risk. Nearly 60% of the variation in the gap between model and data across the 329 bonds is accounted for by vintage effects interacted with a Junior tranche dummy. Junior tranches offered high yields only in the first two years of the program. But from 2015 until 2021 junior bond spreads were too low according to our model. Mezzanine bond spreads have been too high every year. Among junior tranches, those associated with high-LTV mortgage pools were most strongly overpriced. Nearly 70% of the variation in the gap can be accounted for by vintage effects interacted with the Junior tranche and the high-LTV deal dummies.

The main reason for the high model-implied credit spreads on junior CRT bonds is that the junior bonds carry the most default risk, and default risk is highly priced in the model. The average CRT investor appears to be underestimating the differential credit risk exposure.[3] The model's wedge between junior and mezzanine bond spreads is also influenced by prepayment risk. Prepayments return mezzanine tranche principal before junior tranche principal, shortening the duration of mezzanine bonds and hence the length of time that CRT investors are exposed to credit risk. The complexity of the cash-flow waterfall (and changes therein across vintages) makes it difficult to assess the differential impact of prepayment on the various tranches.

With our model in hand, our second main exercise is to price the credit risk on the entire pool of conforming mortgages issued by the GSEs. The resulting credit spread is the fair compensation investors would require to bear all of the credit risk in a given mortgage vintage, not just the risk that the GSEs lay off through the CRT bonds.[4] The fair model-implied credit spread can be compared to the observed guarantee fee (or g-fee) that the GSEs charge mortgage borrowers. The observed average g-fee increased from around 20 bps before the GFC to around 40 bps in 2013 and 50-60 bps between 2014 and 2022. It currently stands at around 65 bps. Put differently, over the past 15 years, the GSEs have tripled the insurance premium they charge mortgage borrowers for taking on credit risk. Is this increase justified? Did it correct a woeful pre-GFC underpricing? Or have the GSEs overdone it, generating excess profits at the expense of homeowners, potentially harming homeownership by unduly increasing the cost of mortgage credit?

Our model shows that the fair compensation for credit risk is 37.8 bps. This compares to an observed average credit risk component of the guarantee fee of 38.7 bps.[5] In other words, our third main result is that the large increase in g-fees post-GFC was justified. The difference between the model-implied and observed g-fee is a mere 1 bp over the full sample. The gap has increased to 9.5 bps in 2023 and to 9.7 bps in 2024. We show that about two-thirds of the model-implied g-fee is a credit risk premium and the remaining one-third is compensation for expected loss. We also compute the fair cost of bearing catastrophic losses, which we define as all losses above a 5% loss rate. This tail risk compensation is 14.5 bps of the overall 37.8 bps credit risk compensation, or about 38%.

Our last set of results concerns the cross-sectional pricing of mortgage credit risk. In the wake of the GFC, the GSEs introduced loan-level price adjustments (LLPAs) that charge high-risk borrowers more than low-risk borrowers. The main differentiators in the LLPAs are the LTV ratio and the FICO score of the borrower. A major change to the LLPA pricing grid in May 2023 reduced LLPAs for riskier (low-FICO) borrowers and increased them for more creditworthy (high-FICO) borrowers. Did the 2023 reform increase the alignment between cross-sectional variation in credit risk and LLPAs or worsen it? Our fourth main result is that the pre-2023 reform LLPA grid overcharged low-FICO borrowers and undercharged high-FICO borrowers. The May 2023 reform

[3]In the data, hedge funds tend to buy the junior tranches while insurance companies tend to purchase the mezzanine tranches. Hedge funds may be targeting a higher return and may be unable to leverage up the mezzanine tranches enough to manufacture the desired return they earn on junior bonds.

[4]CRT bonds do not cover all mortgage debt, even in the later years of the program, and the GSEs do not lay off all of the risk even on the mortgages in the collateral pool underlying a CRT bond. In particular, the GSEs hold on to the tail credit risk.

[5]The remaining 20 basis points of the observed g-fee is a general tax of 10 bps and an administrative fee for running the program of 10 bps.

significantly reduced the overpricing of low-FICO borrowers, and made the overall pricing closer to actuarially fair. However, the reform over-corrected for high-LTV borrowers with LTV ratios below 85% who went from being over-charged to being significantly under-charged. As such the reform may have created a new problem of encouraging high mortgage leverage.

The answers to these questions have taken on renewed urgency in light of the policy momentum to end conservatorship and re-privatize the GSEs.[6] In this emerging regime, where the GSEs may once again operate as private firms—but still benefit from a government guarantee—the pricing of mortgage credit risk becomes a first-order policy concern. If taxpayers are to be compensated for the credit risk that results from the government backstop, it is crucial to determine what a fair compensation (guarantee fee) would be, lest we convert public risk into private profit. This requires a rigorous, market-consistent estimate of the price of mortgage default risk. Our model offers a framework for answering that question—at the level of individual mortgage borrowers, CRT tranches, and entire mortgage vintages.

Related Literature Our study is closely related to the still limited literature on credit risk transfer programs at the GSEs. The descriptive study of Finkelstein et al. (2018) discusses the initial evolution of the CRT program from 2013 until 2017, and documents that CRTs materially reduced the government's exposure to credit loss without disrupting the liquidity of the mortgage market.

An important precursor to our work is Golding and Lucas (2020), who examine CRT security pricing using market data. Their exploratory analysis raises important questions but does not construct a structural model to jointly price interest rate, prepayment, and default risk as we do here. O'Neill (2022) develops a valuation model for individual CRT bonds, but employs a reduced-form default model calibrated directly to observed CRT tranche prices.[7] Flanagan (2025) calibrates his model to match the observed spreads of subordinate CRT tranches, and then use it to price tail risk and infer the guarantee fee. In contrast, we estimate the stochastic discount factor from Treasury spreads, corporate bond spreads, and housing returns and then use it to jointly price subordinate tranches, tail risk, and g-fee. Our headline results are consistent with Flanagan (2025), as we both find that the GSEs are not subsidizing borrowers.

Kim et al. (2024) examine the distributional impact of g-fees on purchase mortgage originations. As a by-product of the analysis, they construct a model that produces LLPAs across FICO–LTV bins while accounting for borrower default and prepayment. They find cross-subsidization from low- to high-LTV borrowers and from high- to low-FICO score groups. We find the same cross-subsidization in the LTV dimension but the reverse pattern in the FICO dimension. As discussed

[6] While earlier efforts during the first Trump administration stalled after the 2020 election, the second Trump Administration has revived them with new intensity. Recent equity market activity reflects growing investor expectations of a transition to private ownership: between June 2024 and June 2025, the share prices of Fannie Mae and Freddie Mac rose by 600% and 700%, respectively. News broke of President Trump's intention to privatize the GSEs in August 2025. See `https://fticommunications.com/the-prospects-of-privatization-for-fannie-mae-and-freddie-mac-in-2025/`.

[7] O'Neill (2022) assumes that tranches are fully wiped out in default, while our approach models default at the individual mortgage level and then aggregates to the bond or pool level.

in Section 7, the discrepancy between the results stems from differences in the pricing model, in how default rates are calibrated, and in how much idiosyncratic risk there is in the default risk within a given FICO-LTV bin.

Gete et al. (2025) uses daily CRT bond spreads to assess investors' views on mortgage default during the Covid-19 period, showing that the forbearance programs in the CARES Act raised expected default rates. They infer a market-implied g-fee from CRT spreads, whereas our paper derives credit spreads within an asset pricing model that jointly prices prepayment and default risk, without relying on CRT prices as inputs. Capponi et al. (2021) constructs a reduced-form model to assess the impact of mortgage forbearance on refinancing activities, and finds that unemployment risk is a significant driver of default-induced prepayments. Our study directly links unemployment risk to borrowers' liquid wealth in a structural default model.

Our model assumes that the same financial intermediaries who trade corporate credit risk and interest rate risk are the marginal investors in the CRT bond market, in the spirit of the intermediary-based asset pricing literature (Adrian et al., 2014; He et al., 2017) and (Gabaix et al., 2007; Diep et al., 2021; Fuster et al., 2024) in the mortgage-backed securities market.

Our paper is also related to the large literature that studies the origins of the U.S. subprime mortgage crisis (Gerardi et al., 2008; Mian and Sufi, 2009; Davis and Van Nieuwerburgh, 2015; Favilukis et al., 2017). These studies highlight the connection between foreclosures and aggregate house price fluctuations.

A different stream of literature has explored the link between mortgage risk and the GSEs (Acharya et al., 2010, 2011, 2013; Elenev et al., 2016; Gupta, 2022). For example, Acharya et al. (2010) caution that government coinsurance programs could devolve into another form of subsidy. We quantify the tail risk—the portion of risk held by the government. Acharya et al. (2010) also hypothesize that some investors may prefer uninsured, higher-yield mortgage securities and bear the risk themselves. Our results confirm this, showing that investor risk-bearing capacity is stronger than expected, as GSEs can issue even the riskiest CRT bonds on favorable terms. Finally, while McGowan and Nguyen (2023) document that the GSEs do not price regional credit risk, our analysis (which does not incorporate regional variation) finds that they underprice high-LTV borrowers' credit risk and overprice low-LTV borrowers' credit risk. Frame et al. (2015) discuss hypothetical scenarios where the GSEs are replaced by a private system. Under these circumstances, the market expects the government to intervene under extreme scenarios. We provide a consistent framework for pricing mortgage tail risk.

We contribute to the literature on prepayment risk (e.g., Chernov et al., 2018; Boyarchenko et al., 2019; Diep et al., 2021; Fuster and Vickery, 2015), which primarily focuses on MBS and mortgage lending, by instead studying the implications of prepayment risk for CRT bond pricing. We also add to the default risk modeling literature (e.g., Deng et al., 2000; Demyanyk and Van Hemert, 2011; Campbell and Cocco, 2015; Ganong and Noel, 2020a) by calibrating and testing a double-trigger default model that we show to closely match historical borrower default rates. Our modern asset pricing framework jointly captures the common risk factors underlying both prepayment and default risk. We apply a structural model of mortgage credit with thousands of borrowers in the tradition of Merton (1974). (Longstaff and Rajan, 2008) applies a reduced-form approach in decomposing the spreads of the Collateralized Debt Obligations into idiosyncratic and systematic components. We model the systematic and idiosyncratic components structurally simulated from a VAR process that allow us to directly link market prices of risk in the housing and bonds market into the borrower's default decisions and the valuation of the collateral at default resolution. Our modeling of credit risk combines insights from the double trigger theory, which says that borrowers' defaults are triggered by negative income and negative home equity (e.g., Ganong and Noel, 2020b; Fuster and Willen, 2017). We find that the double trigger model fits the historical default patterns well, adding additional evidence for its validity.

2 Institutional Details

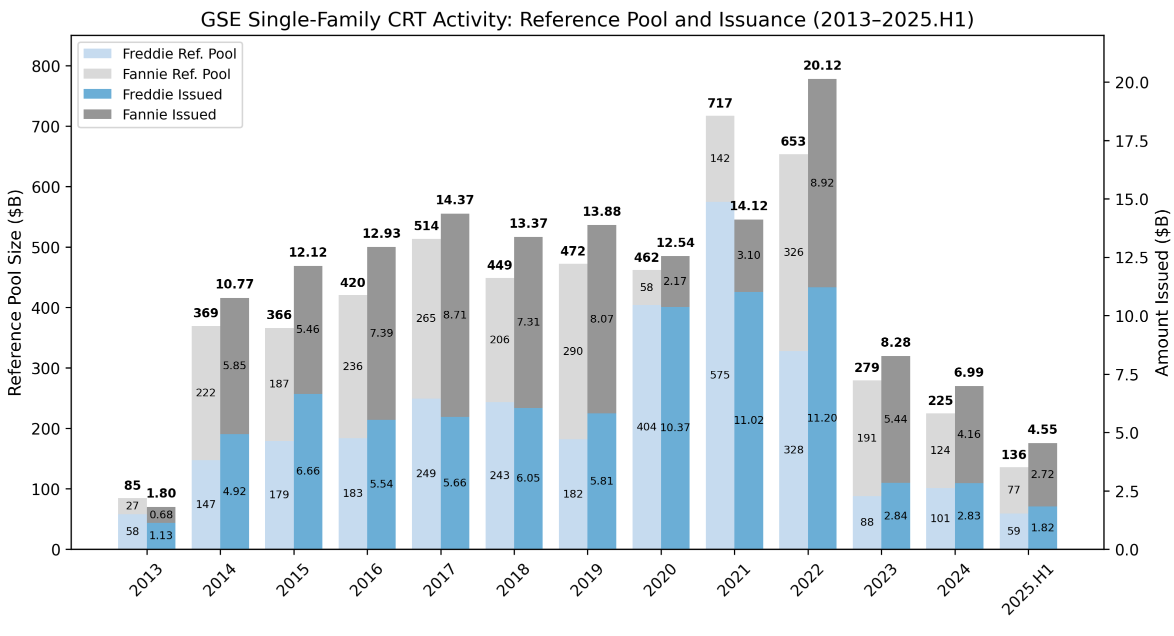
Following the collapse and government bailout of Fannie Mae and Freddie Mac, the GSEs were taken into conservatorship in September of 2008. With taxpayers on the hook for the credit risk in the conforming mortgages purchased by the GSEs, the Federal Housing Financing Agency (FHFA)–the regulator of the GSEs–took three major sets of actions to reduce the taxpayers exposure: increase the guarantee fee, differentiate the pricing of risk through the introduction of the loan-level price adjustments (LLPAs), and launch the credit risk transfer (CRT) program. The former increased the compensation the GSEs received for bearing the credit risk from mortgage borrowers from about 25 basis points per year on each dollar of mortgage principal in 2011 to 65 basis points in 2025. The latter used some of that guarantee fee revenue to pay the private sector to take over some of that credit risk. What started as a pilot program in 2013 grew steadily thereafter, with peak CRT issuance in 2021 and 2022 covering the credit risk on $1 trillion of mortgage principal originated. CRT origination volumes declined in 2023–25 as mortgage origination itself declined due to higher mortgage rates.

There are two types of CRT programs: security issuance and reinsurance transactions. The former account for about 2/3 of CRT issuance while the latter account for 1/3. We focus on the CRT securities. Freddie Mac's securities program is called the Structured Transfer of Agency Credit Risk (STACR) program; Fannie Mae's securities program is called the Connecticut Avenue Securities (CAS) program. Between 2013 and the middle of 2025, $75.9 billion in STACRS were issued covering the credit risk on a reference pool of $2.795 trillion. Over that same period $70.0 billion in CAS were issued covering the credit risk on a reference pool of $2.351 trillion. Hence, the GSEs have sold $145 billion in CRT bonds to investors on $5.15 trillion in mortgage principal. Figure 1 shows the evolution of the STACR and CAS programs between 2003 and 2025.H1. The total coverage ratio over the sample period is approximately 2.8% of reference pool principal.[8] We

[8] As noted, the GSEs have also laid off credit risk to the private insurance sector in the form of explicit reinsurance through the Agency Credit Insurance Structure (ACIS) program at Freddie Mac and the Credit Insurance Risk Transfer (CIRT) program at Fannie Mae. Between 2013 and the end of 2023, the GSEs issued $60.1 billion in reinsurance deals on a $1.545 trillion mortgage reference pool. Data on the reinsurance deals for 2024 and 2025.H1 are not yet available. Combining CRT securities and reinsurance deals, over $205 billion dollars of CRT instruments have been issued on a reference pool of over $6.7 trillion in single-family mortgage principal.

focus on the STACR program, the largest of the two securities' programs. Since the CAS program is structured nearly identically, all our conclusions should apply equally to the CAS program. Our model should also be very useful for pricing the reinsurance deals which are structured similarly to the CRT bonds. Finally, there are similar CRT securities and reinsurance programs on the multi-family side of Fannie Mae and Freddie Mac's business. Again, our analysis should be a useful starting point there as well.

Figure 1: CRT Issuance



Notes: Bars show the annual volume of GSE single-family CRT activity. Each year has two bars: the left bar, plotted against the left axis, reports the size of the reference pool of mortgage collateral underlying the CRT bonds (in $ billions). The right bar, plotted against the right axis, shows the amount of CRT bonds issued (in $ billions). Each bar stacks the amounts of Fannie Mae and Freddie Mac activity. The data source is the Urban Institute's *Housing Finance at a Glance June 2025 Chartbook.*

The CRT program works as follows. Every deal delineates a specific reference pool of mortgages, originated during the same short window of time, with principal balances below the conforming loan limit, underwritten following GSE guidelines. The loan tape of each reference pool is publicly available. The losses on these reference pools that are transferred to CRT investors. There are two types of reference pools. The "DNA" deals refer to reference pools of mortgages with loan-to-value ratios below 80%. The "HQA" deals contain mortgages with LTV ratios between 80% and 97%. In each deal, multiple bonds, known as tranches, are issued. Each tranche has an attachment point, the pool loss rate above which the tranche starts taking losses, and a detachment point, the pool loss rate above which the tranche is fully wiped out. For example, the 2017-DNA1 STACR deal, issued on February 7, 2017, is the first STACR deal of 2017 (hence the 1), refers to a pool of $33.97 billion of 30-year fixed-rate single-family mortgages with LTV ratios below 80%, originated prior to the cutoff date December 15, 2016. Four tranches are sold to CRT investors with attachment and detachment points in parentheses: the B-2 (0.0%–0.5%), B-1 (0.5%–1.0%), M-2 (1.0%–2.55%), and M-1 (2.55%–3.75%) tranches. Losses on the mortgage pool accrue to these tranches in reverse order of seniority: the first 0.5% of losses are transferred to the investors in the B-2 tranche. If losses exceed 0.5%, the B-2 tranche is fully wiped out. Losses above this level get allocated next to the B-1 tranche investors. If losses exceed 1%, both the B-2 and B-1 tranches are wiped out and the M-2 tranche begins to take losses. Finally, if losses are above 2.55%, the M-1 bondholders begin to take losses. If pool losses exceed 3.75%, all CRT bonds sold to the public are wiped out. The B tranches are referred to as junior bonds, the M tranches as mezzanine bonds. Combined, they are referred to as the subordinate tranches. Losses above 3.75% accrue to the A-H tranche. This tranche, known as the senior tranche, is not sold to the public. In other words, the GSEs (currently, the taxpayers) hold the tail or catastrophic loss risk. In this example, 3.75% is referred to as the credit enhancement (CE) of the senior tranche. The legal maturity of these four CRT tranches is July 29, 2029, or 12.5 years after issuance.

Table A.1 lists all tranches included in our study, which is the universe of STACR bonds. The number of tranches sold in a deal ranges between 3 and 5, the attachment and detachment points of tranches vary across deals, as do the CE levels.[9]

One important, maybe underappreciated aspect of CRT bond pricing is the role of mortgage prepayment. Scheduled mortgage payments, i.e., regular principal amortization, repays the principal on senior and CRT tranches pro rata before June 2018, but in order of seniority within the CRT tranche universe. After June 2018, these scheduled mortgage payments only reduce the principal on the CRT bonds if certain conditions are met. Unscheduled principal payments (prepayments and principal recovery after a loss event) are initially just paid to the senior tranche. Once certain conditions are reached, they also begin to reduce the principal on the CRT bonds. But within the CRT bond universe, they are paid again in strict order of seniority. The upshot of this is that the amount of tranche principal that is at-risk of credit loss shrinks over the life of the bond, and more rapidly for the mezzanine tranches than for the junior tranches. What may legally be a 12.5-year credit risk exposure may end up only exposing CRT investor capital for 2 years, if prepayments are unusually fast. The duration of credit risk exposure is shorter than the legal maturity and shorter for the M than for the B tranches. This effect naturally should get reflected in the compensation CRT investors demand for bearing the credit risk.

In return for taking on mortgage credit risk, CRT bond investors receive a monthly interest payment equal to a short-term reference rate (30-day SOFR since 2021, before LIBOR) plus a fixed tranche spread set at issuance. It is this tranche spread that our asset pricing model solves for, and which we will compare to the observed counterpart. For example, the 2017-DNA1's M-1 tranche

[9] CE levels range from 3.0% in the very first deal to 6.4% in the 2015-HQA2 deal. There are other differences across deals. In some deals, the first-loss piece is not sold to the CRT investors but retained by the GSEs. In the 2013 and 2014 CRT vintages, a 180-day mortgage delinquency automatically triggered a loss event, and the loss severity was fixed at a pre-determined schedule. Starting in 2015, losses were allocated to CRT investors when the loan got resolved and the severity was the actual realized loss rate. Starting in 2023, some senior (A) tranches are created and sold to the public. We discuss all of this and more in great detail below. In addition to the senior A tranche and sometimes the first-loss tranche, the GSEs retain a 5% vertical slice of the B and M tranches, selling the remaining 95%. This vertical risk retention is to align incentives between GSEs and CRT investors.

had a spread of 1.2%, the M-2 tranche a spread of 3.25%, the B-1 a tranche spread of 4.95% and the B-2 bond a spread of 10.0%.[10]

The investors in CRT bonds are a mix of insurance companies, hedge funds, asset managers, broker-dealers, REITs, and pension and sovereign wealth funds. Consistent with the spirit of our model, these investors typically also invest in fixed income securities such as treasuries and corporate bonds. We will assume that the prices of Treasuries and corporate bonds are informative about the sources of risk that are present in the agency credit risk market as well. We will additionally consider house price and labor income risk which directly affect the risk of mortgage delinquency.

According to a 2021 FHFA report (Federal Housing Finance Agency, 2021), the GSEs had paid out \$12 billion in interest payments to CRT bond investors and \$3.0 billion in premium payments to CRT reinsurance counterparties. In return, the GSEs had received approximately \$0.03 billion in loss recuperation via investor bond principal write-downs and \$0.02 billion via reinsurance reimbursements. The discrepancy between a \$0.05 billion benefit and a \$15 billion cost to the GSEs has raised concerns over the effectiveness of the CRT program.

In the remainder of this paper, we provide a structural model to price the universe of CRT tranches issued since inception. Our model speaks to the question of whether the large payments to hedge funds, asset managers, and private insurers were justified given the risks that were transferred.

3 Asset Pricing Model

We set up a flexible dynamic asset pricing model in the spirit of the dynamic affine term structure literature (Singleton, 2002; Ang and Piazzesi, 2003), applied in other contexts (Lustig et al., 2013; Gupta and Van Nieuwerburgh, 2021; Koijen et al., 2024). The model is designed and calibrated to match key features of government bond yields and corporate bond returns, alongside key macroeconomic variables relevant to the housing market.

3.1 VAR

Time is discrete and indexed by months. We construct a N-dimensional aggregate state vector where $N = 7$:

$$\tilde{z}_t := (r_t, ys_t, cr_t, cs_t, l_t, h_t, k_t)',$$

where $r_t = y_t(12)$ represents the 1-year constant maturity Treasury (CMT) yield; $ys_t = y_t(120) - y_t(12)$ represents the spread between 10- and 1-year CMT yields; cr_t represents the monthly log corporate credit return on BBB-rated bonds; cs_t represents credit spread between BBB-rated corporate and 10-year CMT bonds; h_t represents the monthly log change in house prices, k_t represents the monthly log change in aggregate disposable per capita income, and l_t represents the monthly layoff rate in the labor market.

[10] One appealing design feature of CRT bonds, from the perspective of the GSEs, is that there is no counterparty risk. The investor pays the GSEs \$100 upfront, held in escrow, and receives the interest rate on the principal that remains at risk, as well as principal repayments due to scheduled and unscheduled principal payments on the underlying mortgage pool. Losses get subtracted as soon as they are realized from the remaining CRT bond principal.

We express the state vector as $\tilde{z}_t = \mu + z_t$, where the demeaned state variable z_t follows a first-order VAR process that describes the dynamics of the economy:

$$z_t = \Phi z_{t-1} + \Sigma^{1/2} \varepsilon_t, \tag{1}$$

where the shock vector ε_t is independently and identically distributed as standard normal, i.e., $\varepsilon_t \sim \mathcal{N}(0, I)$. The matrix $\Sigma^{\frac{1}{2}}$ is the Cholesky decomposition of the residual covariance matrix Σ, a lower-triangular matrix. The ordering of variables in z_t dictates that each orthogonal ε shock affects only itself and the variables ordered after it in the VAR.

Guided by the data, we impose the following zero restrictions on the companion matrix:

$$\Phi = \begin{bmatrix} \phi_{r,r} & \phi_{r,ys} & 0 & 0 & 0 & 0 & 0 \\ \phi_{ys,r} & \phi_{ys,ys} & 0 & 0 & 0 & 0 & 0 \\ \phi_{cr,r} & \phi_{cr,ys} & 0 & \phi_{cr,cs} & 0 & 0 & 0 \\ \phi_{cs,r} & \phi_{cs,ys} & 0 & \phi_{cs,cs} & 0 & 0 & 0 \\ \phi_{l,r} & \phi_{l,cs} & 0 & \phi_{l,cs} & \phi_{l,l} & 0 & 0 \\ \phi_{h,r} & \phi_{h,ys} & 0 & \phi_{h,cs} & 0 & \phi_{h,h} & 0 \\ \phi_{k,r} & \phi_{k,ys} & 0 & \phi_{k,cs} & 0 & 0 & \phi_{k,k} \end{bmatrix},$$

We do not make assumptions on $\Sigma^{\frac{1}{2}}$ and label its entries as follows:

$$\Sigma^{1/2} = \begin{bmatrix} \sigma_{r,r} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{ys,r} & \sigma_{ys,ys} & 0 & 0 & 0 & 0 & 0 \\ \sigma_{cr,r} & \sigma_{cr,ys} & \sigma_{cr,cr} & 0 & 0 & 0 & 0 \\ \sigma_{cs,r} & \sigma_{cs,ys} & \sigma_{cs,cr} & \sigma_{cs,cs} & 0 & 0 & 0 \\ \sigma_{l,r} & \sigma_{l,ys} & \sigma_{l,cr} & \sigma_{l,cs} & \sigma_{l,l} & 0 & 0 \\ \sigma_{h,r} & \sigma_{h,ys} & \sigma_{h,cr} & \sigma_{h,cs} & \sigma_{h,l} & \sigma_{h,h} & 0 \\ \sigma_{k,r} & \sigma_{k,ys} & \sigma_{k,cr} & \sigma_{k,cs} & \sigma_{k,l} & \sigma_{k,h} & \sigma_{k,k} \end{bmatrix}$$

3.2 Idiosyncratic Risk

We introduce idiosyncratic income and house price risk for each mortgage borrower i. Denote by H_t^i the house price level and by h_t^i the house price growth rate (log price change) of borrower i.

Denote by K_t^i the level of disposable income of borrower i and by k_t^i its growth rate (log change):

$$
\begin{aligned}
H_t^i &= H_0^i \cdot \exp\left(\sum_{s=1}^{t} h_s^i\right) && (2)\\
K_t^i &= K_0^i \cdot \exp\left(\sum_{s=1}^{t} k_s^i\right), && (3)\\
h_t^i &= e_6' \cdot \left(\mu + \Phi z_{t-1} + \Sigma^{1/2}\varepsilon_t\right) + \sigma_{h,i}\varepsilon_t^{h,i} && (4)\\
k_t^i &= e_7' \cdot \left(\mu + \Phi z_{t-1} + \Sigma^{1/2}\varepsilon_t\right) + \sigma_{k,i}\varepsilon_t^{k,i}, && (5)
\end{aligned}
$$

where $e_i = (0, \dots, 0, \underbrace{1}_{i\text{-th position}}, 0, \dots, 0)^\top$ denotes the i-th canonical vector. Each borrower's house price growth and income growth consists of an aggregate risk and an idiosyncratic risk component, where $(\varepsilon_t^{h,i}, \varepsilon_t^{k,i})$ denote the idiosyncratic shocks and $(\sigma_{h,i}, \sigma_{k,i})$ are the corresponding shock volatilities.

Individual borrowers start out with (H_0^i, K_0^i, A_0^i) at the time of mortgage origination, where A_0^i denotes initial financial wealth. Mortgage borrowers face the risk of job loss. Let $L_{i,t} \sim \text{Bernoulli}(l_t)$ denote the layoff indicator for borrower i in period t, where $l_t \in [0,1]$ is the aggregate layoff rate (an element of the state vector). Conditional on being laid off ($L_{i,t} = 1$), re-employment in the next period occurs with probability *jfr*: $R_{i,t} \,|\, L_{i,t} = 1 \;\sim \text{Bernoulli}(jfr)$, with draws independent across borrowers and periods.

3.3 Stochastic Discount Factor

The log of the stochastic discount factor $M_t = \exp(m_t)$ is given by the following process:

$$m_{t+1} = -\frac{y_t(1)}{12} - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1}, \tag{6}$$

where the market prices of risk is affine in the state vector:

$$\Lambda_t = \Lambda_0 + \Lambda_1 z_t, \tag{7}$$

and the annual yield on a one-month risk-free bond $y_t(1)$ follows the process:

$$y_t(1) = \delta_0 + \delta_1 e_1' z_t + \delta_2 e_2' z_t. \tag{8}$$

Hence, $y_t(1)$ is an affine function of the one-year bond yield r_t and the term-structure slope ys_t, which are the first and second components of the state vector, respectively. It then follows from the risk-neutral pricing equation $\mathbb{E}_t\left[M_{t+1}\exp\left(\frac{y_t(1)}{12}\right)\right] = 1$ that $-\mathbb{E}_t[m_{t+1}] - \frac{1}{2}Var_t[m_{t+1}] = \frac{y_t(1)}{12}$.

The dynamics of the demeaned VAR under the risk neutral measure follow the same process as in (1) but with risk-neutral companion matrix $\Phi^{\mathbb{Q}} = \Phi - \Sigma^{1/2}\Lambda_1$. The risk-neutral mean of the state vector is $\mu^{\mathbb{Q}} = \mu - \Sigma^{1/2}\Lambda_0$.

We explain in detail below how to identify the various coefficients in (7) and (8). This leads us to impose the following zero restrictions on the market price of risk coefficients:

$$\Lambda_0' = \begin{pmatrix} \Lambda_0^r & \Lambda_0^{ys} & \Lambda_0^{cr} & 0 & 0 & \Lambda_0^h & 0 \end{pmatrix}$$

$$\Lambda_1 = \begin{pmatrix} 0 & \Lambda_1^{r,ys} & 0 & 0 & 0 & 0 & 0 \\ \Lambda_1^{ys,r} & 0 & 0 & 0 & 0 & 0 & 0 \\ \Lambda_1^{cr,r} & \Lambda_1^{cr,ys} & 0 & \Lambda_1^{cr,cs} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Lambda_1^{h,r} & \Lambda_1^{h,ys} & 0 & \Lambda_1^{h,cs} & 0 & \Lambda_1^{h,h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The model has four sources of priced, aggregate risk, associated with the orthogonal shocks to the short rate, the slope of the term structure, credit returns, and house price growth. All four market prices of risk fluctuate over time with the short rate and the slope. The market price of corporate credit risk and house price risk additionally fluctuate with the credit spread, while the market price of house risk additionally depends on lagged house price growth.

We recursively estimate these coefficients in our main asset pricing exercises.

3.4 Treasury and Corporate Bond Pricing

We want to impose that our asset pricing model prices government bonds and corporate bonds accurately.

Starting with government bonds, a standard result in this class of models is that the log yield at time t of a Treasury bond with maturity s is affine in the state vector:

$$\frac{y_t(s)}{12} = -\frac{A_s}{s} - \frac{B_s'}{s} z_t, \tag{9}$$

where the coefficients A_s and B_s satisfy the following difference equations:

$$\begin{aligned} A_{s+1} &= -\frac{\delta_0}{12} + A_s + \frac{1}{2}(B_s)' \Sigma (B_s) + (B_s)' \left(\mu - \Sigma^{\frac{1}{2}} \Lambda_0\right) \\ (B_{s+1})' &= (B_s)' \left(\Phi - \Sigma^{\frac{1}{2}} \Lambda_1\right) - \frac{(\delta_1, \delta_2, 0, \ldots, 0)'}{12}, \end{aligned} \tag{10}$$

with initial conditions $A_0 = 0$ and $B_0 = (0, 0, \ldots, 0)'$.

Since the one-year yield and the difference between the ten- and one-year yields, both expressed as annual yields, are the first and second elements of the VAR, respectively, we have that

$$y_t(120) = (e_1 + e_2)' \mu + (e_1 + e_2)' z_t.$$

By (9), we also have

$$\frac{y_t(120)}{12} = -\frac{A_{120}}{120} - \frac{B'_{120}}{120} z_t.$$

Matching up these two equations, we obtain

$$A_{120} = -10(e_1 + e_2)'\mu, \quad B'_{120} = -10(e_1 + e_2)' \tag{11}$$

Similar conditions for the one-year bond imply

$$A_{12} = -e'_1\mu, \quad B'_{12} = -e'_1 \tag{12}$$

the above conditions guarantee that the model matches the observed average slope of the term structure.

The expected excess return on the 10-year bond, also known as the bond risk premium, is defined as:

$$\begin{aligned} \mathbb{E}\left[r_{t,t+1}(120)|z_t\right] - \frac{y_t(1)}{12} &= \mathbb{E}\left[p_{t+1}(119)\right] - p_t(120) - \frac{y_t(1)}{12} \\ &= \left(A_{119} - A_{120} - \frac{\delta_0}{12}\right) + \left(B'_{119}\Phi - B'_{120} - \left(\frac{\delta_1}{12}e_1 + \frac{\delta_2}{12}e_2\right)'\right) z_t \end{aligned} \tag{13}$$

where $r_{t,t+1}$ is the return from purchasing the ten year treasury bond at month t and selling it at month $t+1$, and $p_t(s)$ is the time t log price of a bond maturing in s months. According to the affine pricing model, $p_t(s) = A_s + B'_s z_t$. The second equality above applies this formula.

Next, we turn to pricing the return on the BBB-corporate bond index. From the first-order condition, we have that:

$$\mathbb{E}_t\left[cr_{t+1} - \frac{y_t(1)}{12}\right] + \frac{1}{2}Var_t\left[cr_{t+1}\right] = -Cov_t\left[cr_{t+1}, m_{t+1}\right] \tag{14}$$

The left-hand side is pinned down by the VAR, while the right-hand side can be obtained using the SDF expression in (6).

$$\begin{aligned} \mathbb{E}_t\left[cr_{t+1} - \frac{y_t(1)}{12}\right] + \frac{1}{2}Var_t\left[cr_{t+1}\right] &= \mu_{cr} + \phi_{cr,r}r_t + \phi_{cr,ys}ys_t + \phi_{cr,cs}cs_t \\ &\quad -\frac{\delta_0}{12} - \frac{\delta_1}{12}r_t - \frac{\delta_2}{12}ys_t \\ &\quad +\frac{1}{2}\left(\sigma^2_{cr,r} + \sigma^2_{cr,ys} + \sigma^2_{cr,cr}\right) \\ -Cov_t\left[cr_{t+1}, m_{t+1}\right] &= \Lambda^r_0\sigma_{cr,r} + \Lambda^{ys}_0\sigma_{cr,ys} + \Lambda^{cr}_0\sigma_{cr,cr} + \left(\Lambda^{ys,r}_1\sigma_{cr,ys} + \Lambda^{cr,r}_1\sigma_{cr,cr}\right) r_t \\ &\quad + \left(\Lambda^{r,ys}_1\sigma_{cr,r}\Lambda^{cr,ys}_1\sigma_{cr,cr}\right) ys_t + \Lambda^{cr,cr}_1\sigma_{cr,cr}cs_t \end{aligned}$$

where r_t, ys_t, cs_t in the above expressions are understood to be demeaned. It follows that:

$$\mu_{cr} - \frac{\delta_0}{12} + \frac{1}{2}\left(\sigma^2_{cr,r} + \sigma^2_{cr,ys} + \sigma^2_{cr,cr}\right) = \Lambda_0^r \sigma_{cr,r} + \Lambda_0^{ys}\sigma_{cr,ys} + \Lambda_0^{cr}\sigma_{cr,cr} \tag{15}$$

$$\phi_{cr,r} - \frac{\delta_1}{12} = \Lambda_1^{ys,r}\sigma_{cr,ys} + \Lambda_1^{cr,r}\sigma_{cr,cr} \tag{16}$$

$$\phi_{cr,ys} - \frac{\delta_2}{12} = \Lambda_1^{r,ys}\sigma_{cr,r} + \Lambda_1^{cr,ys}\sigma_{cr,cr} \tag{17}$$

$$\phi_{cr,cs} = \Lambda_1^{cr,cs}\sigma_{cr,cr} \tag{18}$$

3.5 Housing Returns

Finally, we insist on pricing the return on housing. The return on housing consists of a capital gain return plus an income return. The latter is also known as the net rental yield. We assume that the net rental yield is constant and equal to nry per month. Since this is the return on an individual house, part of the return variance is the idiosyncratic component $\sigma^2_{h,i}$

$$\mathbb{E}_t\left[h^i_{t+1} + nry - \frac{y_t(1)}{12}\right] + \frac{1}{2}Var_t\left[h^i_{t+1}\right] = -Cov_t\left[h^i_{t+1}, m_{t+1}\right] \tag{19}$$

The left-hand side is pinned down by the VAR, while the right-hand side can be obtained using the SDF expression in (6).

$$\begin{aligned}
&\mathbb{E}_t\left[h_{t+1} + nry - \frac{y_t(1)}{12}\right] + \frac{1}{2}Var_t\left[h_{t+1}\right] = \mu_h + nry + \phi_{h,r}r_t + \phi_{h,ys}ys_t + \phi_{h,cs}cs_t + \phi_{h,h}h_t \\
&-\frac{\delta_0}{12} - \frac{\delta_1}{12}r_t - \frac{\delta_2}{12}ys_t + \frac{1}{2}\left(\sigma^2_{h,r} + \sigma^2_{h,ys} + \sigma^2_{h,cr} + +\sigma^2_{h,cs} + \sigma^2_{h,l} + \sigma^2_{h,h} + \sigma^2_{h,i}\right) \\
&-Cov_t\left[h_{t+1}, m_{t+1}\right] = \Lambda_0^r\sigma_{h,r} + \Lambda_0^{ys}\sigma_{h,ys} + \Lambda_0^{cr}\sigma_{h,cr} + \Lambda_0^h\sigma_{h,h} + \left(\Lambda_1^{ys,r}\sigma_{h,ys} + \Lambda_1^{cr,r}\sigma_{h,cr} + \Lambda_1^{h,r}\sigma_{h,h}\right)r_t \\
&+\left(\Lambda_1^{r,ys}\sigma_{h,r} + \Lambda_1^{cr,ys}\sigma_{h,cr} + \Lambda_1^{h,ys}\sigma_{h,h}\right)ys_t + \left(\Lambda_1^{cr,cs}\sigma_{h,cr} + \Lambda_1^{h,cs}\sigma_{h,h}\right)cs_t + \Lambda_1^{h,h}\sigma_{h,h}h_t
\end{aligned}$$

where r_t, ys_t, cs_t in the above expressions are understood to be demeaned. It follows that:

$$\mu_h + nry - \frac{\delta_0}{12} + \tfrac{1}{2}\sum_{j\in\{r,ys,cr,cs,l,h,i\}}\sigma^2_{h,j} = \Lambda_0^r\sigma_{h,r} + \Lambda_0^{ys}\sigma_{h,ys} + \Lambda_0^{cr}\sigma_{h,cr} + \Lambda_0^h\sigma_{h,h}, \tag{20}$$

$$\phi_{h,r} - \frac{\delta_1}{12} = \Lambda_1^{ys,r}\sigma_{h,ys} + \Lambda_1^{cr,r}\sigma_{h,cr} + \Lambda_1^{h,r}\sigma_{h,h}, \tag{21}$$

$$\phi_{h,ys} - \frac{\delta_2}{12} = \Lambda_1^{r,ys}\sigma_{h,r} + \Lambda_1^{cr,ys}\sigma_{h,cr} + \Lambda_1^{h,ys}\sigma_{h,h}, \tag{22}$$

$$\phi_{h,cs} = \Lambda_1^{cr,cs}\sigma_{h,cr} + \Lambda_1^{h,cs}\sigma_{h,h}, \tag{23}$$

$$\phi_{h,h} = \Lambda_1^{h,h}\sigma_{h,h}. \tag{24}$$

3.6 Mortgage Dynamics

The borrower takes out a 30-year fully-amortizing fixed-rate mortgage, the standard mortgage product in the United States. The maturity is $T = 360$ months. Let N_0^i be the mortgage principal and $r_0^{m,i}$ the annual rate on the 30-year fixed-rate mortgage of borrower i at loan origination. Let $N_0 := \sum_{i=1}^{I} N_0^i$ be the aggregate notional of all mortgage loans originated across borrowers in that loan vintage. Absent mortgage default and prepayment, the monthly mortgage payment, including interest and principal amortization, is

$$C^i := \frac{\frac{r_0^{m,i}}{12} N_0^i}{1 - \left(1 + \frac{r_0^{m,i}}{12}\right)^{-T}}.$$

The unpaid principal balance (UPB) evolves according to:

$$N_t^i = N_0^i \times \frac{\left(1 + \frac{r_0^{m,i}}{12}\right)^{T} - \left(1 + \frac{r_0^{m,i}}{12}\right)^{t}}{\left(1 + \frac{r_0^{m,i}}{12}\right)^{T} - 1},$$

absent prepayment or default.

We model the average 30-Year fixed-rate mortgage rate as an affine function of the 10-year constant-maturity Treasury bond:

$$r_t^m = r_0^m + b \cdot (r_t + ys_t - r_0 - ys_t), \tag{25}$$

where r_0^m is the unconditional mean of the mortgage rate and b its sensitivity to the demeaned 10-year T-bond yield.

The borrower's (after-tax) disposable income at mortgage origination is K_0^i. We can back out this income from the observed debt-to-income ratio at the time of mortgage origination d_0^i: $K_0^i = \frac{C^i}{(1+\tau^{inc})d_0^i}$.

We describe the evolution of the borrower's liquid financial wealth, A_t^i, next. This liquid financial wealth will play an important role in mortgage default. The borrower can either be laid off at time t or not laid off. If she is not laid off, has positive financial wealth at t, and sufficient disposable income to make both her periodic mortgage payment and pay for consumption necessities ($0.8 \cdot K_t^i - C^i > 0$), then she is a saver. Her time-t assets are growing at the risk-free rate of interest and she adds savings from disposable income in the current period. We assume that 20% of disposable income is required for consumption necessities. We assume that the borrower is able to save 13.5% of her disposable income after the mortgage payment, which is the average savings rate out of post-mortgage disposable income observed in the data.

$$A_{t+1}^i = \begin{cases} \left(\frac{r_t}{12}+1\right)A_t^i + \underbrace{0.135\cdot(K_t^i - C^i)}_{\text{savings from DPI}} & \text{if not laid off, } 0.8\cdot K_t^i - C^i \geq 0, A_t^i \geq 0 \\ \left(\frac{r_t}{12}+1\right)A_t^i + \left(0.8\cdot K_t^i - C^i\right) & \text{if not laid off, } 0.8\cdot K_t^i - C^i < 0, A_t^i \geq 0 \\ \left(\frac{r_t+ccs}{12}+1\right)A_t^i + 0.135\cdot\left(K_t^i - C^i\right) & \text{if not laid off, } 0.8K_t^i - C^i \geq 0, -12K_t < A_t^i < 0 \\ \left(\frac{r_t+ccs}{12}+1\right)A_t^i + \left(0.8\cdot K_t^i - C^i\right) & \text{if not laid off, } 0.8K_t^i - C^i < 0, -12K_t < A_t^i < 0 \\ \left(\frac{r_t}{12}+1\right)A_t^i - C^i & \text{if laid off, } A_t^i \geq 0 \\ \left(\frac{r_t+ccs}{12}+1\right)A_t^i - C^i & \text{if laid off, } -12K_{t_{i,l}}^i < A_t^i < 0 \end{cases}$$

If she is not laid off, starts the period with positive assets, but has insufficient income to pay for consumption necessities and the mortgage, then her assets go down by $0.8\cdot K_t^i - C^i < 0$.

If she is not laid off but starts the period in debt ($A_t^i < 0$), she pays an interest rate on her debt that exceeds the risk-free interest rate by a annual consumer credit spread of ccs. If the income net of consumption necessities and mortgage payment is positive, she still accumulates 13.5% of post-mortgage disposable income in savings. Otherwise, her income go down by $0.8\,K_t^i - C^i$. We assume that consumer debt must not exceed twelve months of income ($-12K_{t_{i,l}}^i < A_t^i$), a standard unsecured borrowing limit. Whenever a borrower exceeds this limit, we force her to sell the house and repay the mortgage principal. If the borrower has positive home equity, this liquidation does not result in a loss to the lender.

If the borrower is laid off, we assume her income drops to the level that is just sufficient to pay for consumption necessities. her mortgage must be paid from prior savings.

Finally, if the borrower is laid off at t and was already in debt at t, her debt accumulates further by the amount of the mortgage payment, assuming it stays within the borrowing limit, where we have used $K_{t_l}^i$ to denote the income at the time of the layoff. Otherwise, the mortgage is liquidated.

3.6.1 Mortgage Default

Following the consensus in the literature, we adopt the double trigger theory of mortgage default. A borrower i defaults if the equity in the home becomes negative—after considering a fire sale discount fd_t^i and a sales commission sc,—and has insufficient liquid assets to pay the mortgage. Define the default event as:

$$t_{i,def} := \inf\{t \leq T : \{A_t^i \leq C^i\} \cap \{(1-fd_t^i)(1-sc)H_t^i \leq N_t^i\}\}. \tag{26}$$

We set $\tau_t^i = 1$ if borrower i has defaulted by time t, and $\tau_t^i = 0$ if borrower i has not defaulted by time t and the borrowing constraint on unsecured consumer credit has not been hit. We have $P(\tau_t^i = 1) = P(t_{i,def} \leq t)$ and $P(\tau_t^i = 0) = 1 - P(\tau_t^i = 1)$.

When a loan defaults, it takes time before the loan gets resolved. Denote the resolution time by $t_{i,res} > t_{i,def}$. The loan resolution results in (i) a loss realization $l_{t_{i,res}}^i$ and (ii) a partial recovery of the unpaid principal balance $R^i_{t_{i,resf}}$. The implementation section below discusses how realized loss and recovery are computed in the context of CRT transactions.

3.6.2 Mortgage Prepayment

Modeling mortgage prepayment accurately is important for our purposes as a prepayment extinguishes the mortgage and hence the associated credit risk born by CRT investors.

We define the rate incentive of borrower i, $\delta^i_t := r^{m,i}_0 - r^{m,i}_t$, as the difference between the mortgage rate borrower i obtained at loan origination and the mortgage rate she could obtain on a new 30-year fixed-rate mortgage issued in the current period t.

As shown by the literature on mortgage prepayment, the rate incentive is far and away the most important determinant of mortgage prepayment. We model the monthly prepayment rate, also known as the single month mortality rate, on a mortgage (pool) with coupon (interest rate) c as a function of the rate incentive:

$$\Gamma^i(\delta^i_t) := \frac{L^c}{1 + e^{-\alpha^c(\delta^i_t - A)}}. \tag{27}$$

The functional form gives rise to the well-known S-shaped prepayment curve, with low but positive prepayment rates at negative prepayment incentives, steeply rising prepayment rates for modestly positive rate incentives, and flattening prepayment rates at high rate incentives.

An individual mortgage borrower prepays each period with probability given by (27). Let ϵ^i_t denote the mortgage prepayment random variable of borrower i: $\epsilon^i_t = 1$ if the borrower has prepaid by time t and $\epsilon^i_t = 0$ if the borrower i has not prepaid by time t.

3.7 Credit Risk Transfer Bond Pricing

Our main exercise is to price the Credit Risk Transfer (CRT) bonds. As explained in Section 2, CRT bonds are structured credit products that pass some of the losses on an underlying mortgage loan pool from the GSEs to private investors. The underlying mortgage pool consists of standard 30-year fixed-rate mortgages originated during a specific time window.

The cumulative loss rate as of time t on a pool of I mortgages originated at time 0, expressed as a share of the initial aggregate loan principal N_0, is:

$$L_t := \sum_{i=1}^{I} \frac{l^i_{t_{i,res}} \mathbf{1}_{t_{i,res} \leq t}}{N_0}, \tag{28}$$

Principal repayments on a mortgage come in two flavors: scheduled principal payments from amortization and unscheduled principal payments, which consist of prepayments and recoveries from default resolutions. The cumulative scheduled and unscheduled principal repayments as of time t on a pool of I mortgages originated at time 0, expressed as a share of the initial aggregate loan principal N_0, is:

$$F_t^S = \sum_{i=1}^{I} \frac{(N_0^i - N_t^i)\mathbf{1}_{\tau_t^i=0,\epsilon_t^i=0}}{N_0}, \tag{29}$$

$$F_t^U = \sum_{i=1}^{I} \frac{R_{t_{def}^i}^i \mathbf{1}_{\tau_t^i=1,\epsilon_t^i=0} + N_0^i \mathbf{1}_{\tau_t^i=0,\epsilon_t^i=1}}{N_0}, \tag{30}$$

where scheduled principal payments come from borrowers who have neither prepaid nor defaulted at of time t ($\mathbf{1}_{\tau_t^i=0,\epsilon_t^i=0} = 1$), and unscheduled principal payments come from borrowers that have prepaid as of time t ($\mathbf{1}_{\tau_t^i=0,\epsilon_t^i=1} = 1$) or have defaulted ($\mathbf{1}_{\tau_t^i=1,\epsilon_t^i=0} = 1$).

In each CRT vintage, multiple bonds (tranches) are issued (sold to investors) with different attachment points b, the pool loss rate above which the tranche begins to take losses, and detachment points b', the pool loss rate at which the tranche is wiped out. We recall that the most senior tranche (called A-H) is always retained by the GSEs. This tranche begins to take losses when the pool loss rate exceeds a level CE (credit enhancement), equal to the detachment point of the most senior sold tranche. Denote the share of the pool's UPB that is sold to CRT bond investors by M.

It is crucial to understand how the pool's realized losses, scheduled principal payments, and unscheduled principal payments are allocated to the various tranches.

Losses are distributed in reverse order of seniority. The most junior tranche (usually the B-2 tranche) suffers credit losses first. Only after it has been fully wiped out do losses accrue to the next most junior bond (the B-1 tranche), etc. The cumulative loss rate incurred by the $[b, b']$ tranche as of time t is:

$$L_t^{[b,b']} := \min\left\{\max\left\{L_t - b, 0\right\}, b' - b\right\} \tag{31}$$

Principal recoveries from default resolutions are allocated entirely to the senior A-H tranche. Given the size of the A-H tranche (typically well over 90% of the total), the CRT bond investors do not receive any payments that come from recoveries.

Prior to March 2018,[11] scheduled principal payments are allocated to the senior (A-H) tranche and the subordinate tranches pro rata according to the current level of CE. Among the subordinate tranches, scheduled principal payments are allocated sequentially starting with the most senior. The cumulative scheduled principal cash flow paid to the $[b, b']$ CRT tranche as of time t is:

$$F_t^{S,[b,b']} := \min\left\{(M \cdot F_t^S - (M - b'))\mathbf{1}_{M \cdot F_t^S > M - b'}, b' - b\right\} \tag{32}$$

The $[b, b']$ tranche only receives scheduled principal cash flows after all tranches senior to it ($M - b'$) have been fully paid off. It gets fully paid off when $M \cdot F_t^S \geq M - b$.

After March 2018, scheduled principal payments only flow to the subordinate tranches when (i) cumulative loan loss and (ii) recent loan delinquency levels are sufficiently low, and (iii) the current credit enhancement (CE) is sufficiently high. The indicator variable $\mathbf{1}_{delstop,t}$ is one in periods in

[11] This refers to all deals from the first CRT deal, 2013 STACR DN1, to the 2018 STACR DNA1 deal issued in March 2018, based on information in the private placement memoranda.

which the cumulative loss or recent delinquency threshold is breached. The indicator $\mathbf{1}_{CE,t}$ is one when the CE (of the senior tranche) is above the threshold:

$$F_t^{S,[b,b']} := \min\left\{(M \cdot F_t^S - (M - b'))\mathbf{1}_{M \cdot F_t^S > M - b'}\mathbf{1}_{CE,t}(1 - \mathbf{1}_{delstop,t}), b' - b\right\} \tag{33}$$

Unscheduled principal payments are allocated differently from scheduled principal payments.[12] They are initially directed only to the senior A-H tranche until a pre-specified CE threshold is met, as long as delinquency levels are below pre-specified thresholds. Once these thresholds are reached, prepayments are allocated to the senior A-H tranche and the subordinate tranches pro rata according to the current CE level. Among the subordinate tranches, unscheduled principal payments are allocated sequentially, with the most senior investor tranche receiving all prepayments until it is fully paid down. The cumulative unscheduled principal cash flow paid to the $[b, b']$ CRT tranche as of time t is:

$$F_t^{U,[b,b']} := \min\left\{(M \cdot F_t^U - (M - b'))\mathbf{1}_{M \cdot F_t^U > M - b'}\mathbf{1}_{CE,t}(1 - \mathbf{1}_{delstop,t}), b' - b\right\}. \tag{34}$$

Since both scheduled and unscheduled principal payments flow to the senior CRT bonds before they flow to the junior CRT bonds, senior tranches have shorter duration than the junior tranches. Define cumulative principal payments to a tranche as $F_t^{[b,b']} := F_t^{S,[b,b']} + F_t^{U,[b,b']}$.

We define the value-at-risk of tranche $[b, b']$ at time t as:

$$V_t^{[b,b']} := ((b' - b) - L_t^{[b,b']} - F_t^{[b,b']})\mathbf{1}_{L_t^{[b,b']} + F_t^{[b,b']} \leq b' - b} \tag{35}$$

V represents the remaining credit exposure of a tranche, the difference between the original principal of the tranche $(b' - b)$ and the amount of principal repayment and credit losses allocated to that tranche as of time t.

We are now ready to price the CRT bonds. Let T^c be the maturity of the CRT bond expressed in months. The CRT tranche $[b, b']$ pays a spread $ts^{[b,b']}$ over the SOFR (LIBOR) rate sr_t (lr_t), a variable interest rate benchmark. We use sr_t for illustration This interest rate spread is set such that the contract is priced fairly. That is, the present risk-adjusted value of payments on the tranche are equal to the initial investment:

$$\mathbb{E}^{\mathbb{P}}\left[\sum_{s=1}^{T^c}\left(\prod_{t=1}^{s} M_t\right)\left(\underbrace{\frac{ts^{[b,b']} + sr_s}{12}V_s^{[b,b']}}_{Monthly\ Interest\ Payment} + \underbrace{F_s^{[b,b']} - F_{s-1}^{[b,b']}}_{Monthly\ Cash\ Flow}\right) + \left(\prod_{t=1}^{T^c} M_\tau\right)\underbrace{V_{T^c}^{[b,b']}}_{Final\ Credit\ Exposure}\right]$$
$$= \underbrace{b' - b.}_{Initial\ Notional} \tag{36}$$

The CRT investor in this tranche receives principal payments in month s equal to $F_s^{[b,b']} - F_{s-1}^{[b,b']}$ plus

[12] Generally, unscheduled principal payments contain prepayments and default recoveries. Default recoveries exclusively flow to the senior tranche. Hence, for CRT investors, it is prepayments that matter. We discuss this further below in the implementation section.

an interest rate payment of $ts^{[b,b']} + sr_s$ on the remaining value-at-risk to compensate her for bearing credit risk. At the end of the life of the contract, she receives the remaining value-at-risk. In case the losses on the mortgage pool did not reach the level b, the investors receives back all remaining principal. Under the risk-neutral pricing measure $\mathbb{Q}$, which we use in our implementation, this pricing formula becomes:

$$\mathbb{E}^{\mathbb{Q}}\left[\sum_{s=1}^{T^c} \exp\left(-\sum_{t=1}^{s}\frac{y_t(1)}{12}\right)\left(\frac{ts^{[b,b']} + sr_s}{12}V_s^{[b,b']} + (F_s^{[b,b']} - F_{s-1}^{[b,b']})\right) + \exp\left(-\sum_{t=1}^{T^c}\frac{y_t(1)}{12}\right)V_{T^c}^{[b,b']}\right] = b' - b. \quad (37)$$

3.8 Guarantee Fee

The model-implied guarantee fee represents the fair compensation for bearing the credit risk associated with the entire mortgage pool until maturity, taking as given how market participants price interest rate, corporate credit, and house price risks. Unlike CRT bonds, which have shorter maturity than the underlying mortgages ($T^c < T$), the GSEs retain the credit risk exposure over the full life cycle of the loan. As before, principal amortization and prepayments reduce the value that remains at risk.

The pricing formula for the g-fee is a special case of the one for a CRT tranche (37), where the tranche now refers to the entire pool, $[b, b'] = [0, 1]$. Specifically, the model-implied g^{fee} solves:

$$\mathbb{E}^{\mathbb{Q}}\left[\sum_{s=1}^{T} e^{-\sum_{t=1}^{s}\frac{y_t(1)}{12}}\left(\frac{g^{\text{fee}} + y_s(1)}{12}V_s^{[0,1]} + (F_s^{[0,1]} - F_{s-1}^{[0,1]})\right)\right] = 1, \quad (38)$$

where $V_T^{[0,1]} = 0$ since the mortgages are fully amortizing.

4 Estimation

4.1 VAR

We estimate the VAR(1) process using monthly OLS regressions. Appendix A.1 details the data sources. We use the longest available period for the seven variables in the state vector z, which is from January 2001 until February 2025.

For the main asset pricing results below, we estimate the VAR recursively: to price CRT bonds issued in year t (or determine g-fees or LLPAs), we estimate the VAR from the start of the sample in January 2001 until December of year $t - 1$.

Here, for illustration purposes, we present the full-sample VAR estimation results in Table 1. Panel A reports the unconditional mean of the state variables, Panel B the VAR companion matrix, and panel C the innovation volatilities (the Cholesky decomposition of the covariance matrix). The entries in Panels A and C have been multiplied by 100 for readability. Note that the interest rate, yield spread, and credit spread observed from FRED are expressed in annual rates, while the other variables are described in monthly rates.

Table 1: VAR Model Parameters

r	ys	cr	cs	l	h	k
Panel A: Monthly Drift Vector $\boldsymbol{\mu}\times 100$						
0.1568· 12	0.1087· 12	0.4386	0.2086 · 12	1.3718	0.3776	0.3074
Panel B: Transition Matrix Φ						
0.9759	0.0601	0	0	0	0	0
0.0043	0.9571	0	0	0	0	0
0.0540	-0.0705	0	0.2475	0	0	0
0.0110	-0.0265	0	0.9693	0	0	0
0.0165	-0.0565	0	0.0552	0.5258	0	0
-0.0232	-0.0246	0	0.0138	0	0.9381	0
-0.0183	-0.0923	0	-0.0671	0	0	-0.5412
Panel C: Cholesky Factor $\boldsymbol{\Sigma}^{1/2}\times 100$						
0.1919	0	0	0	0	0	0
-0.0713	0.1762	0	0	0	0	0
-0.2926	-0.4677	1.8767	0	0	0	0
-0.0899	-0.0484	-0.0896	0.1410	0	0	0
-0.1709	0.0017	-0.0836	0.1046	0.4307	0	0
0.0122	0.0310	0.0313	-0.0163	0.0077	0.2068	0
0.1409	0.0785	-0.2571	-0.1459	0.0581	-0.0987	1.7184

We note that the volatility of disposable income growth of 1.72% per month in the full sample is more than twice the estimate of 0.75% for the pre-2019 sample. The estimate jumps discretely to 1.01% once the covid-19 year 2020 is added to the sample and to 1.81% once 2021 is added as well, after which it falls back slightly. Similarly, the volatility of the layoff rate over the full sample is 0.43% but only 0.08% in the sample that ends in 2019. It jumps to 0.46% once 2020 is added and remains at 0.46% after 2021 is also added. The other coefficients are more stable over time. A small literature discusses how to best deal with the highly unusual covid-19 shock in the context of VAR estimation (Lenza and Primiceri, 2022; Carriero et al., 2021; Schorfheide and Song, 2021; Primiceri and Tambalotti, 2020). We explored pricing CRT bonds using VAR coefficients estimated on the full sample, on a sample that omits 2020, and recursively. We concluded that the recursive estimation best approximates what market participants would use, based on the information available at each point in time.

4.2 Idiosyncratic Risk

We calibrate the volatility of the monthly idiosyncratic house price growth shock $\epsilon_t^{h,i}$ to $\sigma_{h,i} = 0.0115$, which corresponds to an annual standard deviation of 4%. This number matches the cross-sectional standard deviation of annual MSA-level house price growth during normal times obtained by Sinai and Heathcote (2012, Figure 16).

We set the volatility of the monthly idiosyncratic income shock $\epsilon_t^{k,i}$ to $\sigma_{k,i} = 0.0779$, which corresponds to an annual standard deviation of 27%. This number matches the standard deviation of annual earnings changes for prime-age males in Guvenen et al. (2019).

We set the monthly job finding rate jfr to 20% to match the observed average unemployment spell duration of 5 months given by the Bureau of Labor Statistics.

4.3 Market Prices of Risk

We use Treasury yield data to pin down the coefficients that govern the risk-free rate $(\delta_0, \delta_1, \delta_2)$ and the market prices of risk associated with level and slope risk $(\Lambda_0^r, \Lambda_0^{ys}, \Lambda_1^{r,ys}, \Lambda_1^{ys,r})$. Specifically, we match the six conditions related to A_{12}, A_{120}, B_{12}, B_{120} from (11) and (12). In addition, we insist on matching the average spread between the one-year T-note and the three-month T-bill for an additional moment:

$$-\frac{A_{12}}{12} + \frac{A_3}{3} = \frac{\textit{sample mean of 1Y-3M spread}}{12}.$$

Hence, we have seven moments to exactly identify seven parameters.

Next, we pin down the four market price of risk coefficients associated with credit risk $(\Lambda_0^{cr}, \Lambda_1^{cr,r}, \Lambda_1^{cr,ys}, \Lambda_1^{cr,cr})$ from the four moment conditions (15)-(18).

Finally, we pin down the five market price of risk coefficients associated with house price risk $(\Lambda_0^h, \Lambda_1^{h,r}, \Lambda_1^{h,ys}, \Lambda_1^{h,cs}, \Lambda_1^{h,h})$ from the five moment conditions (20)-(24).

For our main asset pricing exercises, we will estimate all market price of risk coefficients recursively. But to illustrate the model fit, we report below the market prices of risk coefficients estimated on the full sample:

$$\Lambda_0' = \begin{pmatrix} -0.1867 & -0.0286 & 0.1336 & 0 & 0 & 2.7880 & 0 \end{pmatrix}$$

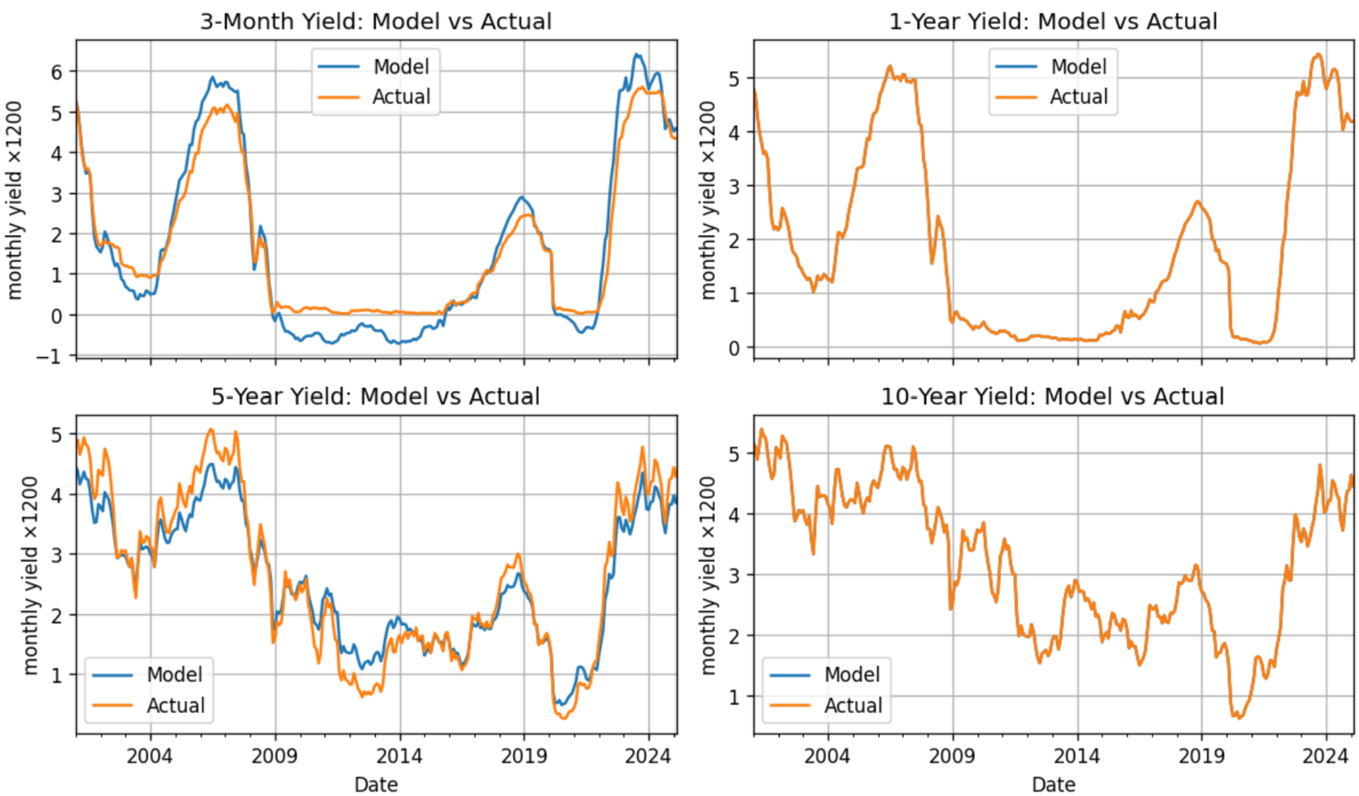
$$\Lambda_1 = \begin{pmatrix} 0 & -45.06 & 0 & 0 & 0 & 0 & 0 \\ -12.75 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.35 & -2.10 & 0 & 13.19 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -55.20 & 6.39 & 0 & 4.68 & 0 & 453.72 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

and the short rate dynamics are given by

$$(\delta_0, \delta_1, \delta_2) = (0.0167, 1.1586, -0.3798).$$

Figure 2 shows the fit of the model for nominal bond yields of various maturities when market prices of risk are estimated on the full sample. The model fits the 1- and 10-year yields perfectly and the 3-month and 5-year government bond yields closely.[13]

Figure 2: Model-Implied vs. Empirical Treasury Yields



Notes: The figure plots the observed and model-implied constant-maturity yields on Treasuries of maturities of three months (top left panel), one year (top right panel), five years (bottom left panel) and ten years (bottom right panel). The sample begins in January 2001. All yields have been multiplied by 1200, i.e., expressed as annual percentages, for readability.

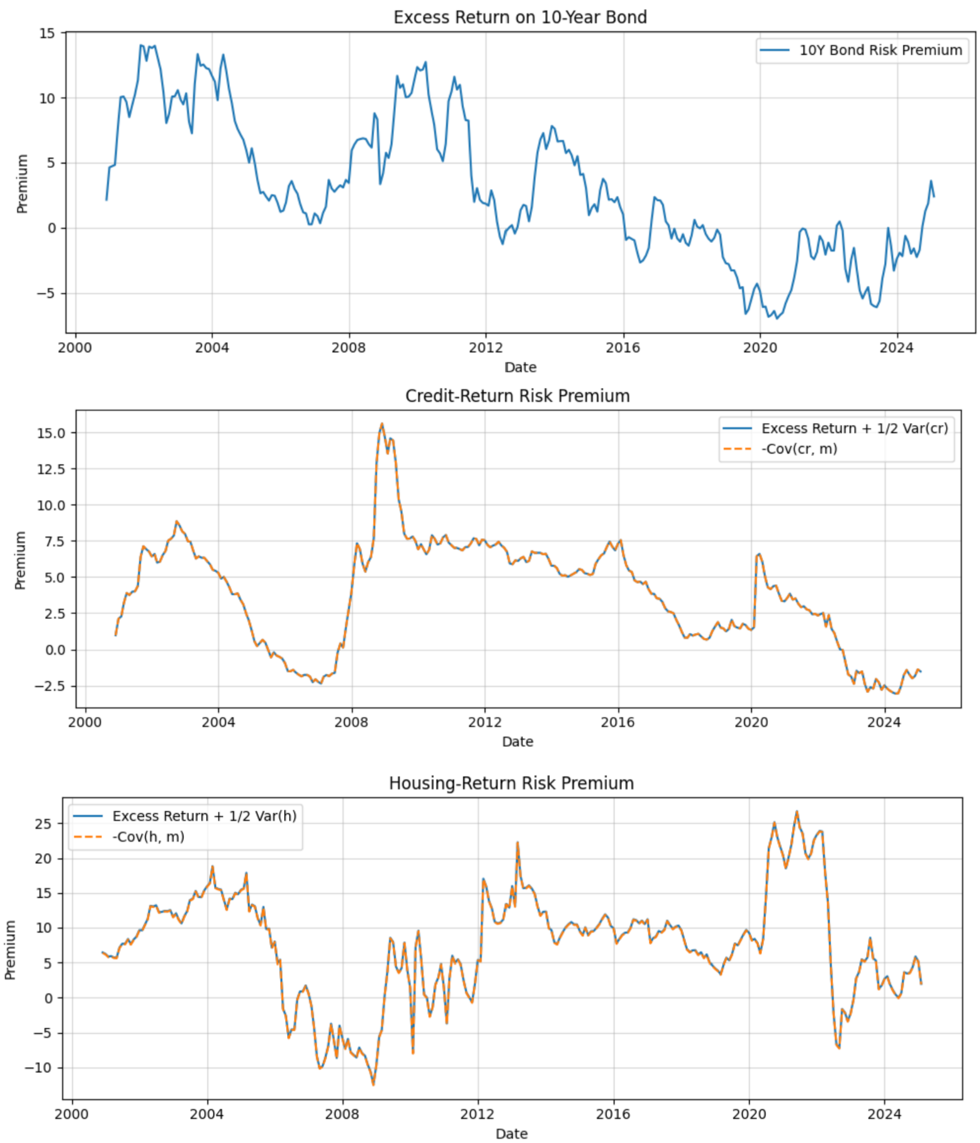
The top panel of Figure 3 plots the time series of the bond risk premium on the ten-year bond in (13). The middle panel of Figure 3 plots the time series of the credit risk premium in (14). It spikes in 2008 and 2020 and reaches its lowest value near the end of the sample.

The bottom panel of Figure 3 plots the time series of the housing risk premium in (19). A key parameter in the model, which has a first-order effect on the average housing risk premium, is the net rental yield nry. Following Eisfeldt and Demers (2015), we set the monthly net rental yield to $nry = 0.045/12$, which is the national average net rental yield for single-family housing in their paper. It has subtracted taxes, maintenance and repair costs, and capital expenditures from the gross yield. The model's annualized housing risk premium averages to 7.51%. The housing risk

[13] The three-month bond yield is at the zero lower bound for parts of the sample. An affine term structure model with Gaussian innovations cannot handle the zero lower bound. The 3-month yield never goes very negative in the model.

premium displays a large increase between 2020 and 2021-22 before falling back sharply.

Figure 3: Nominal Bond Risk Premium



Notes: The top panel plots $\mathbb{E}[r_{t,t+1}(120)] - \frac{y_t(1)}{12}$, the expected excess return on a 10-year bond over the 1-month yield, based on the affine term-structure model; see equation (13). The middle panel plots the model implied credit risk premium in equation (14). It is computed as the expected excess return plus one-half the variance of credit returns (LHS), or alternatively as minus the conditional covariance between the log SDF and the log credit return (RHS). Values are multiplied by 1200 for readability.

4.4 Mortgage Dynamics

We estimate the slope $b = 1.0097$ in (25) using data on the Fixed Rate Mortgage Average in the United States (series MORTGAGE30US) retrieved in FRED from 1989 to 2025.

Using data from FRED (series TERMCBPER24NS), we calibrate the annual spread of the unsecured consumer credit rate over the one-year treasury rate to $ccs = 0.0799$.

We use an income tax rate τ^{inc} of 25% to determine the borrower's initial post-tax income from the debt-to-income ratio.

4.4.1 Mortgage Default Model Estimation

The mortgage default model in (26) has two parameters to pin down. First, the sales commission sc is set to 6%, a standard value for the U.S.

Second, the fire sale (or foreclosure) discount is not directly observed. Campbell et al. (2011) use repeat-sales regressions to estimate that foreclosed homes sell for 27% less on average than comparable non-distressed homes. Gerardi et al. (2015) estimates that the foreclosure discount peaked at nearly 30% in 2009 and declined to approximately 10–15% by 2012 as the housing market stabilized. To capture the time co-variation of the fire sale discount with the state of the housing market, we let the fire sale discount depend on the borrower's house price growth in the previous year:

$$fd_t^i := \min\left\{0, \max\left\{1, 0.2 - \log\left(\frac{H_t^i}{H_{t-36}^i}\right)\right\}\right\}. \tag{39}$$

For example, if a borrower's house lost 15% in value in the previous three years, the fire sale discount would be 35% (0.2+0.15)). For average house price growth around 6% in the past three years, the fire sale discount would be 2%. The coefficient 0.2 in the above expression are obtained as the best fit to the observed historical default rates, plotted in Figure 4.[14] This choice can simultaneously match the large collateral loss rates coming out of the GFC and the minor credit losses after several years of healthy house price growth starting in 2012. Indeed, if the borrower has positive home equity but experiences a negative liquidity shock (e.g. job loss), she can always sell the house instead of going through a foreclosure.

The distribution of liquid assets at the time of mortgage origination is important for the model's ability to generate realistic default behavior. Consistent with this notion, Gerardi et al. (2018) show that liquidity constraints significantly increase the probability of mortgage default, even after controlling for FICO scores and LTV ratios. Gerardi et al. (2018) also find that liquid financial assets correlate positively with FICO scores. Based on this evidence, we assume the following functional form for the initial liquid financial assets for standard loans:

$$A_0^i = K_0^i \cdot \left(2.25 + \max\left\{\left(\frac{\texttt{FICO}^i - 500}{35}\right), 0\right\}\right) \tag{40}$$

This assumption implies that a borrower with a FICO score of 500, which is extremely low, has $A_0^i = 2.25 \cdot K_0^i$, while a borrower with FICO score of 800, which is high, has $A_0^i = 10.82 \cdot K_0^i$. For non-standard mortgage loans, which did not go through the standard underwriting process, we

[14] We conduct a two-dimensional grid search over $\{0.1, 0.15, 0.2\}$ for the intercept coefficient and $\{0, 1, 2, 3, 4, 5\}$ for the number of years.

assume borrowers have $A_0^i = 0.7K_0^i$.[15]

To investigate how well our model fits the historical default rates, we combine loan performance data from Freddie Mac's standard mortgage data set with its non-standard dataset.[16] Figure 4 visualizes the mortgage default rate by loan disposition year. The model fits both the number of loan defaults (top panel) and the UPB of the defaulted loans (bottom panel) well, with much larger default realizations in the years 2008–2010. The vast majority of loans resolved in those years were originated prior to the GFC.

We also study the time series of default rates for a given loan origination year. As an example of a representative pre-GFC vintage, the top panel of Figure 5 plots the cumulative default rate over time of the 2002 loan vintage. The model generates a 1.4% cumulative default rate for this vintage by the year 2024, close to the observed cumulative 1.3% default rate. For the 2007 vintage, one of the all-time riskiest, the middle panel of Figure 5 shows that our model generates a 10.5% cumulative default rate by the year 2024, close to the observed value of 12%. The bottom panel of Figure 5 shows a representative post-GFC cohort, the 2017 vintage. It has a cumulative default rate by the end of 2024 that is around 0.15%. The model replicates the large differences in default risk across vintages.

Figure 6 compares the empirical and model-implied cumulative default probabilities by FICO credit score bin for the 2005 (left panel) and 2012 (right panel) vintages. It illustrates the model's ability to capture the default pattern across credit score segments. Similarly, Figure 7 shows the observed (left) and model-implied (right) default rates by LTV and FICO score for all the vintages combined from 2000 to 2024. The default model does a good job of differentiating between low- and high-risk borrowers. The UPB-weighted root-mean squared error between the default rate in the model and the data across the cells in the grid is 1.4%.

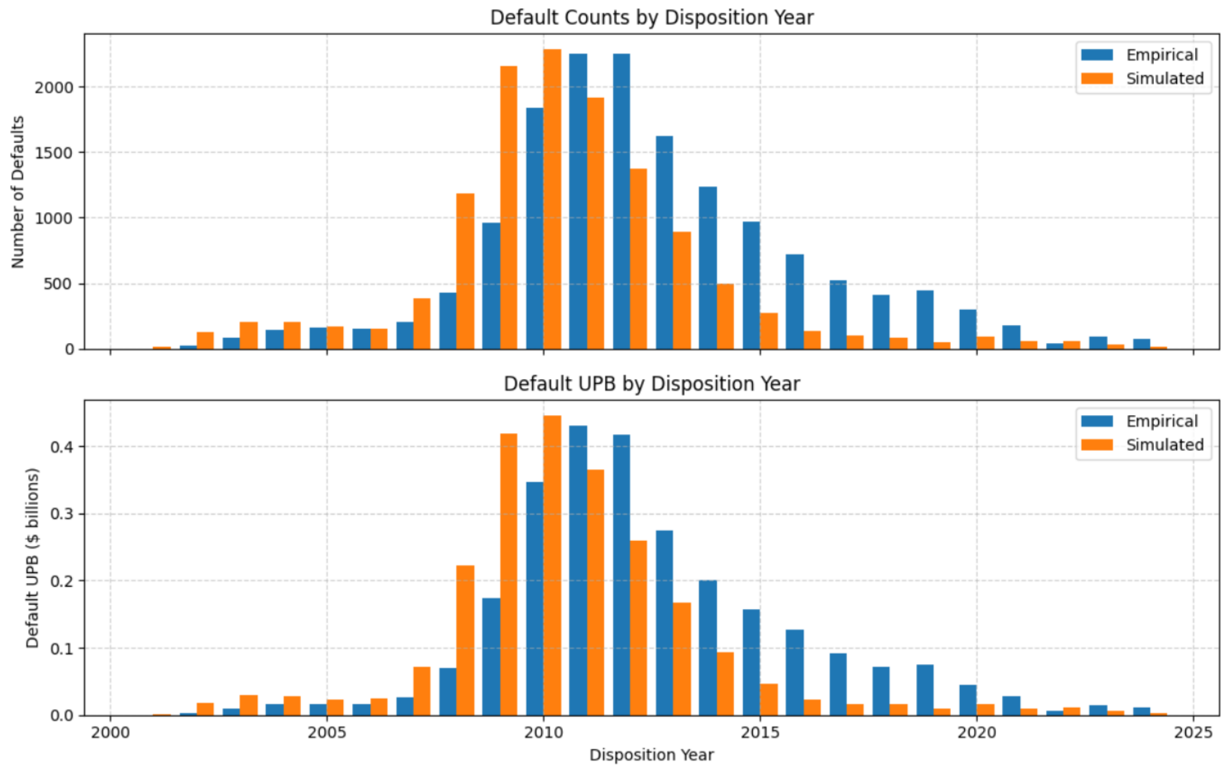
4.4.2 Prepayment Model Estimation

We estimate the prepayment rate function (27) by coupon. Specifically, we group all borrowers with an annual mortgage interest rate between $c\%$ and $(c+1)\%$, and allow (L^c, α^c) to vary by coupon group. We combine coupons 1% and 2% as a single group. Coupons 3%, 4%, 5%, 6%, and 7% are five additional groups. We fix the constant $A = 0.005$. We allow the parameter α^c, which governs the sensitivity of the prepayment rate of a given mortgage pool to the rate incentive, to depend on the age of the mortgage pool (Age, months since origination) and the cumulative fraction of the

[15] Non-standard mortgage loans, such as low- and no-documentation loans, are a pre-GFC phenomenon. The borrower's income and/or assets would not be verified by the lender in such cases. Non-standard mortgages affect the pricing of the CRT bonds insofar that they affect the estimation of the parameters of the mortgage default model.

[16] The data are available from `https://freddiemac.embs.com/FLoan/Data/downloadQ.php`. Since loan default rates were much higher for non-standard loans, it is important to include the non-standard loan data to obtain an accurate view of mortgage default rates. Performance data files for the non-standard loans are available until 2021.Q2. For each mortgage vintage year, we randomly sample 30,000 households with standard loans. We also randomly sample a vintage-specific number of households with non-standard loans, where the number reflects the share of non-standard loan in total loans for each vintage (25% for the vintages from 2000 until 2005, 60% for the vintages from 2006 until 2007, 30% for the 2008 vintage, 25% for the vintages from 2009 to 2010, and 10% for all other vintages). We filter out loans that are not 30-year fixed-rate mortgages. As these shares indicate, the number of non-standard loans fell sharply in the post-GFC era.

Figure 4: Historical vs. Model-Implied Mortgage Default Rates by Disposition Year



Notes: This figure compares model-simulated default outcomes to realized defaults in the Freddie Mac loan-level dataset, comprised of both standard and non-standard mortgages. The top panel plots the number of loans resolved through default in each disposition year. The bottom panel shows the unpaid principal balance (UPB) of those defaulted loans, expressed in billions of U.S. dollars.

pool that has already been prepaid ($Cum\,Prepay$):

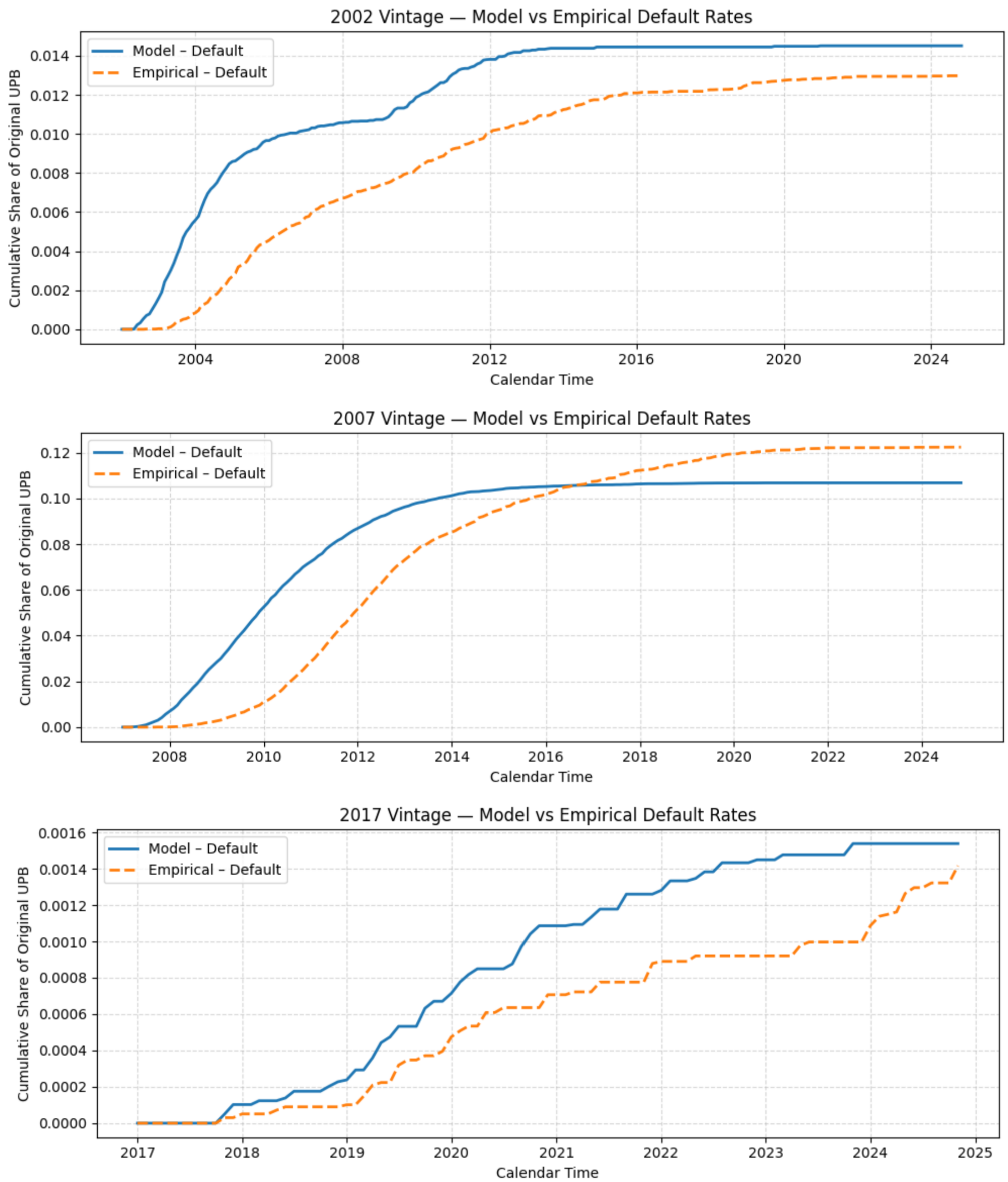
$$\alpha^c(\text{Age}, \text{Cum Prepay}) = \gamma_0^c + \gamma_1^c\text{Age} + \gamma_2^c\text{Cum Prepay}. \tag{41}$$

We also allow L^c to vary by coupon group. L^c is the monthly prepayment rate when the rate incentive is large and positive.

In a given coupon group, there are multiple mortgage pools (vintages) and each pool is observed for multiple months. For each pool-month, we observe the prepayment rate (single month mortality), the age, and the cumulative prepayment fraction. We estimate $(L^c, \gamma_0^c, \gamma_1^c, \gamma_2^c)$ by minimizing the distance between the model-implied and observed prepayment rates. We do this separately for each coupon. The resulting parameter estimates are reported in Table 2. The best UPB-weighted fit for the annualized prepayment rate achieves a mean absolute error of 0.047 and a root mean squared error of 0.060.[17]

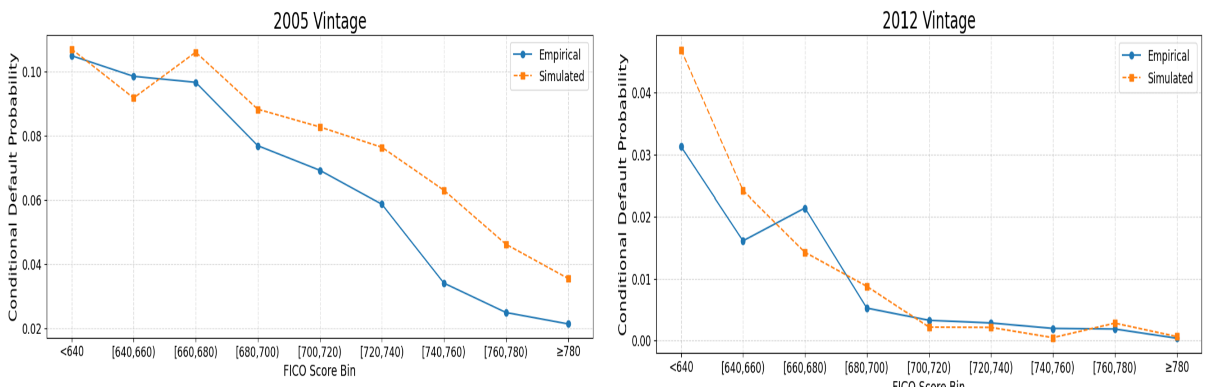
[17] For robustness, we have also experimented with a model that imposes the same coefficients $(L^c, \gamma_0^c, \gamma_1^c, \gamma_2^c)$ across all coupons c. That model results in a somewhat worse fit with a mean absolute error of 0.063 and a root mean

Figure 5: Cumulative Default Rates over Time for Selected Vintages



Notes: This figure compares model-implied and observed cumulative default rates over time for the 2002 mortgage loan vintage (top panel), the 2007 mortgage loan vintage (middle panel), and the 2017 mortgage loan vintage (bottom panel). The observed default rate combines 30,000 randomly-sampled loans from the Freddie Mac standard loan-level mortgage performance dataset with 7500 (18000, 3000) loans from the Freddie Mac non-standard loan-level performance data set for the 2002 (2007, 2017) vintage.

Figure 6: Mortgage Default Rate by Credit Score Group



Notes: This figure compares model-implied and observed cumulative default rates by credit score (FICO) group for the 2005 mortgage loan vintage (left panel) and the 2012 mortgage loan vintage (right panel).

Table 2: Prepayment Function Parameter Estimates

Coupon Group	L^c	γ_0^c	γ_1^c	γ_2^c
1–2	0.009529	0.099982	0.049903	-0.000021
3	0.038748	0.105511	1.540497	-0.000127
4	0.047808	306.615386	-0.459396	0.000253
5	0.038475	149.648308	-0.302620	-0.000310
6	0.044321	158.989545	-0.652514	-0.001732
7	0.091098	681.307838	-9.758352	0.000227

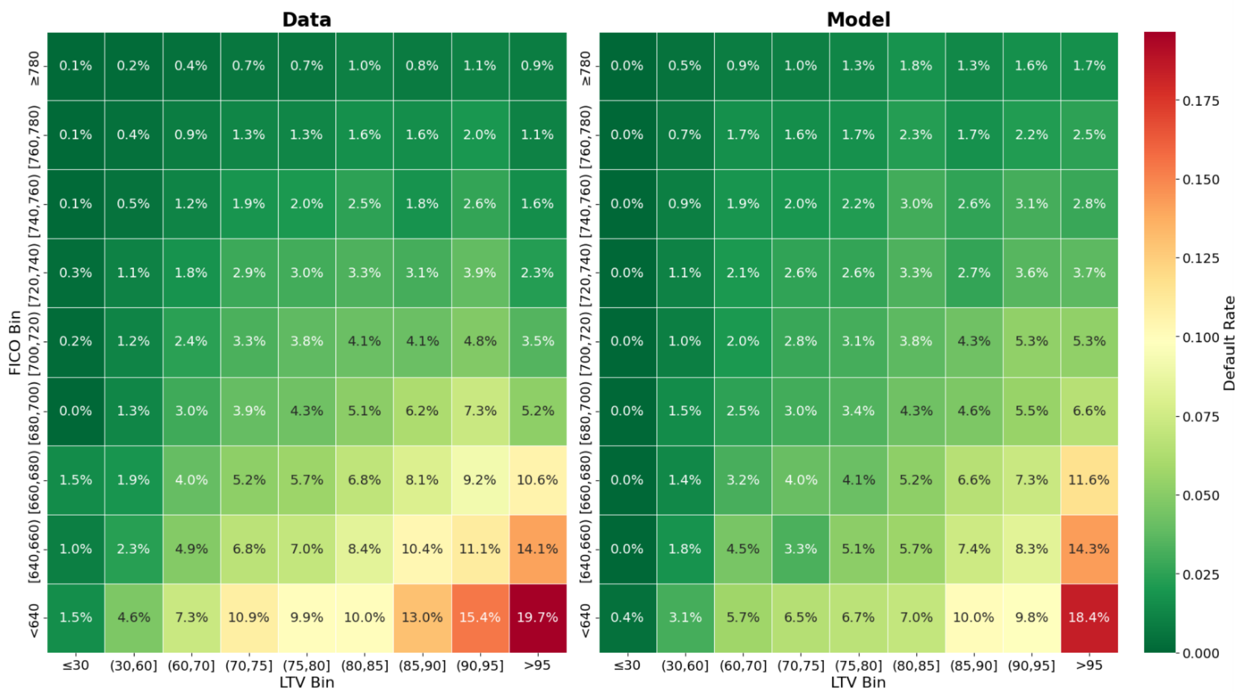
Figure 8 plots the cumulative prepayment rate over time for select vintages. The model captures the fact that some vintages, like the 2002.Q1 and 2019.Q1 vintages prepay quickly, with over 80% of mortgage principal extinguished within five years, while others like the 2013.Q1 vintage prepay more slowly due to different time paths of market mortgage rates. Figure 9 shows the UPB-weighted prepayment rate for all vintages, expressed as an annual number, the so-called conditional prepayment rate or CPR.[18] Our prepayment model fits the data quite well.

Figure 10 compares the model-implied prepayment rate, where we estimate the prepayment function model coefficients using data until year t in order to form the model-implied prepayment rate in year $t + 1$. We start with the sample ending in 2013, then roll the sample forward by one year, and repeat. This out-of-sample prepayment rate still tracks the observed prepayment rate reasonably well. Over the period since 2013, the out-of-sample mean absolute error for the annual prepayment rate is 0.033 and the root mean squared error is 0.051.

squared error of 0.080.

[18] To go from the single month mortality (SMM), which is a monthly number, to the conditional prepayment rate (CPR), which is an annual number, we use: $CPR = 1 - (1 - SMM)^{12}$.

Figure 7: Mortgage Default Rates Across FICO and LTV groups



Notes: This figure compares empirical (left) and simulated (right) conditional default probabilities on a FICO × LTV grid for loans originated from 2000 to 2024. The heatmaps visualize how default risk varies across credit score and loan-to-value groups. The UPB weighted RMSE of the difference between model and data is 1.4 %.

5 CRT Bond Pricing

Before presenting our main CRT pricing results, we discuss several implementation details.

5.1 Implementation Details

Interest Rate Benchmark According to the FHFA Federal Housing Finance Agency (2021), investors receive monthly coupon payments, typically on the 25th day of each month. For the Freddie Mac CRT bonds, the variable-rate payments are tied to the 30-day LIBOR rate until the end of September 2020 and to the 30-day SOFR rate from the start of October 2020 onward. We obtain the 30-day average SOFR and LIBOR rates from FRED. We model the benchmark rate as an affine function of the first two state variables, the one-year Treasury yield r_t and the slope of the yield curve ys_t. To do so, we estimate a regression of one-month LIBOR (SOFR) on a constant and these two state variables for the period 2013.01–2020.12 (2018.01–2025.02). We obtain the coefficient estimates of (0.0052, 0.9466, 0.0259) for the former regression and (0.0011, 0.9086,-0.2638) for the latter regression.[19]

[19]SOFR became actively traded in 2018, and so we prefer to estimate the relationship between SOFR and the two state variables on a slightly longer sample. The results are similar when that relationship is estimated on the

Figure 8: Cumulative Prepayment Rate for Selected Vintages

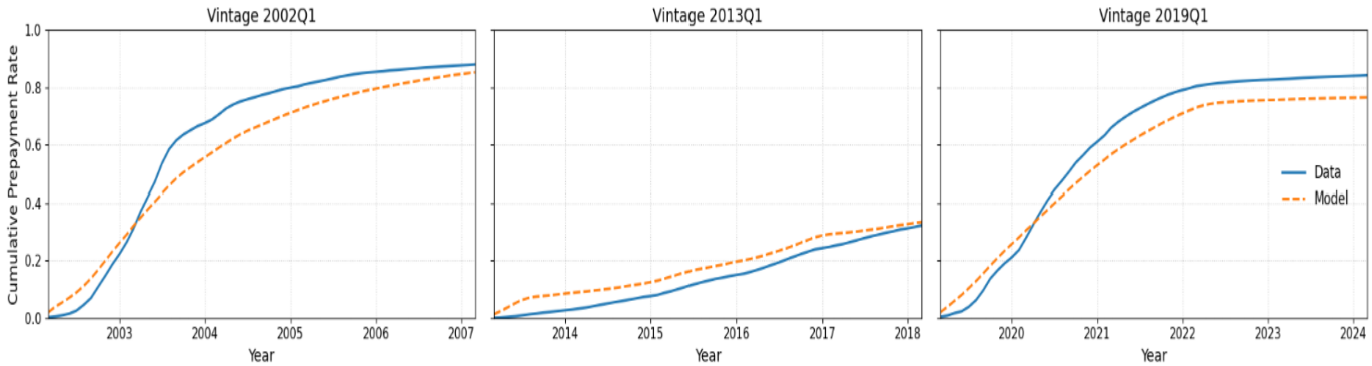
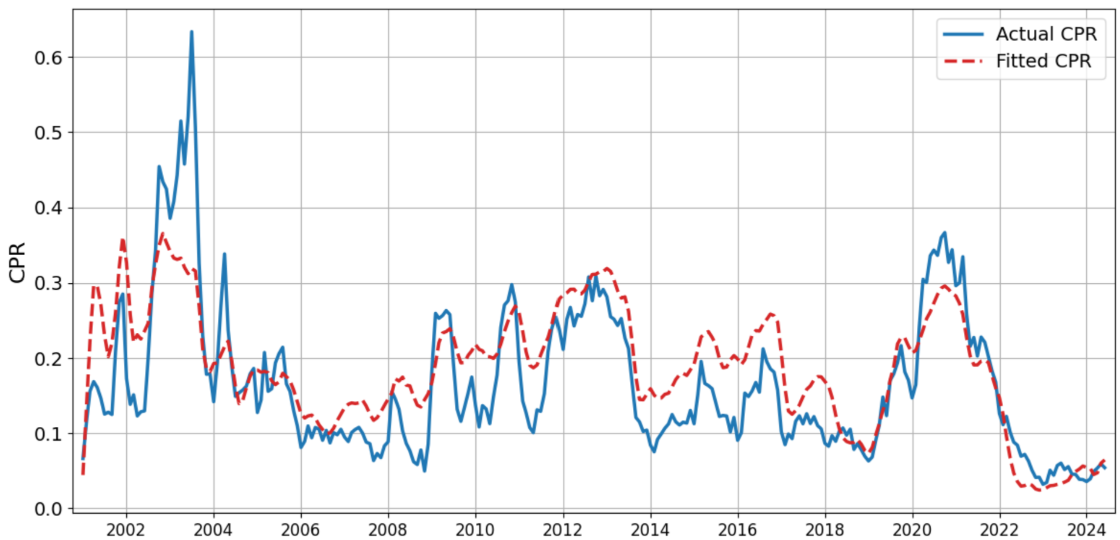


Figure 9: Conditional Prepayment Rate (UPB-weighted average of vintages)

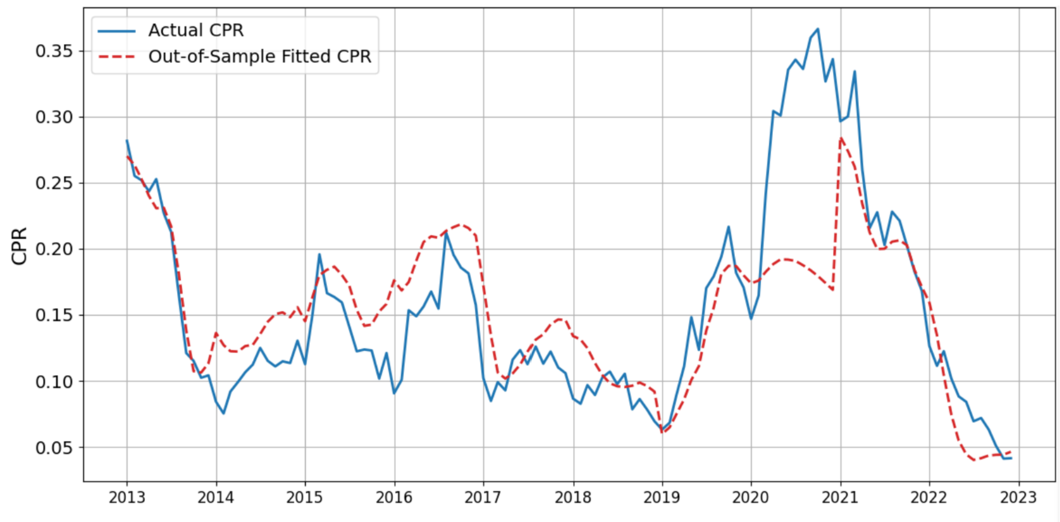


CRT Maturity and Call Option CRT bonds have a legal maturity that varies across deals, ranging from 10 to 30 years. Table A.1 presents the complete list of tranches we study and has a column indicating the legal maturity. The GSEs also have an option to call the CRT bonds early when the UPB falls below 10% of the original UPB to avoid the administrative costs of small-balance deals. Based on the observed clean-up call exercise behavior of Freddie Mac, we assume that the clean-up call is always exercised as soon as possible.[20] Starting in October 2021, STACRs also gave the GSEs a 5-year call option. The legal maturity became 20 years from then onwards. Given that none of these CRTs have reached the 5-year mark, we have no data to discipline the exercise behavior of this 5-year call option. Given that most of the credit risk exposure builds up after five years, we assume that the GSEs will not exercise this option put keep their protection

post-2020.09 sample instead.

[20]Consistent with this assumption, there are no CRT bonds that remain outstanding that have less than 10% of the original UPB.

Figure 10: Out-of-Sample Conditional Prepayment Rate (UPB-weighted).



alive. Of course, should the clean-up call threshold be met before year 5, the CRT will still be called. Accordingly, we set T^c to be the minimum of the legal maturity and the first month that the deal UPB falls below 10% of the origination UPB.

Calculating Losses Absorbed by CRT Investors Next, we explain in more detail how borrowers' defaults impact the loss to CRT tranche holders. The realized loss l_t^i of borrower i is the positive difference between the default costs and default credits:

$$\text{Realized Loss} = \max\{\text{Default Costs} - \text{Default Credits}, 0\}$$

$$\text{Default Costs} = \text{Default UPB} + \text{Accrued Interest} + \text{T \& I} + \text{Legal Fees} + \text{Maintenance Fees}$$

$$\text{Default Credits} = \text{Net Sales Proceeds} + \text{PMI} + \text{Miscellaneous Credits}$$

We define each of these components and explain how they are computed in model and data in Appendix A.2. This includes a detailed discussion of how the delay between loan delinquency and loss resolution is calibrated, and of the way in which private mortgage insurance (PMI) affects credit risk exposure of CRT investors in the high-LTV (HQA) deals.

Fixed Severity Vintages For CRT bonds issued in or after 2015 under the realized loss framework, realized losses and principal recoveries are allocated to the tranches at the time of resolution, as described above.[21] The CRT bonds issued during 2013 and 2014 instead apply a fixed severity schedule. When loans become 180 days delinquent, there is a loss event, and the loss rate that is applied is independent of the actual realized loss but rather follows a fixed severity schedule. In particular, the loss rate is set of 15% for cumulative defaults up to 1%, to 25% severity for cumulative defaults between 1% and 2%, and to 40% severity for defaults above 2%. For the 2013-14

[21]The STACR 2015-DNA1, priced March 2015, was the first deal priced under the actual loss framework.

vintages, there are no default credits, including no PMI credit.

Mortgage Modifications Some delinquent loans receive a modification. The modification may reduce principal or interest or simply extend the maturity, adding the missed payments at the end. When we split cumulative loss rates into those loans that ever received a modification and those that never did, we find that the bulk of losses come from never-modified loans. Figure A.2 shows this break-down for the 2006 vintage, one of the lowest-quality vintages disproportionately subject to post-GFC modification programs. Even this vintage only shows one in six dollars of losses from ever-modified loans. The share is much smaller in the CRT era. This evidence leads us to simply calibrate the model to average losses on all mortgages.

Unscheduled and Scheduled Principal Payments to CRT Investors At each payment date, the GSEs administer tests to determine which types of tranches receive scheduled and unscheduled principal payments and how much cash they receive. These tests relate to (i) the cumulative net loss rate of the collateral pool, (ii) the delinquency rate over the past six months, (iii) the current credit enhancement. We discuss these important cash-flow waterfall details in Appendix A.4.

5.2 Main Results: CRT Tranche Spreads

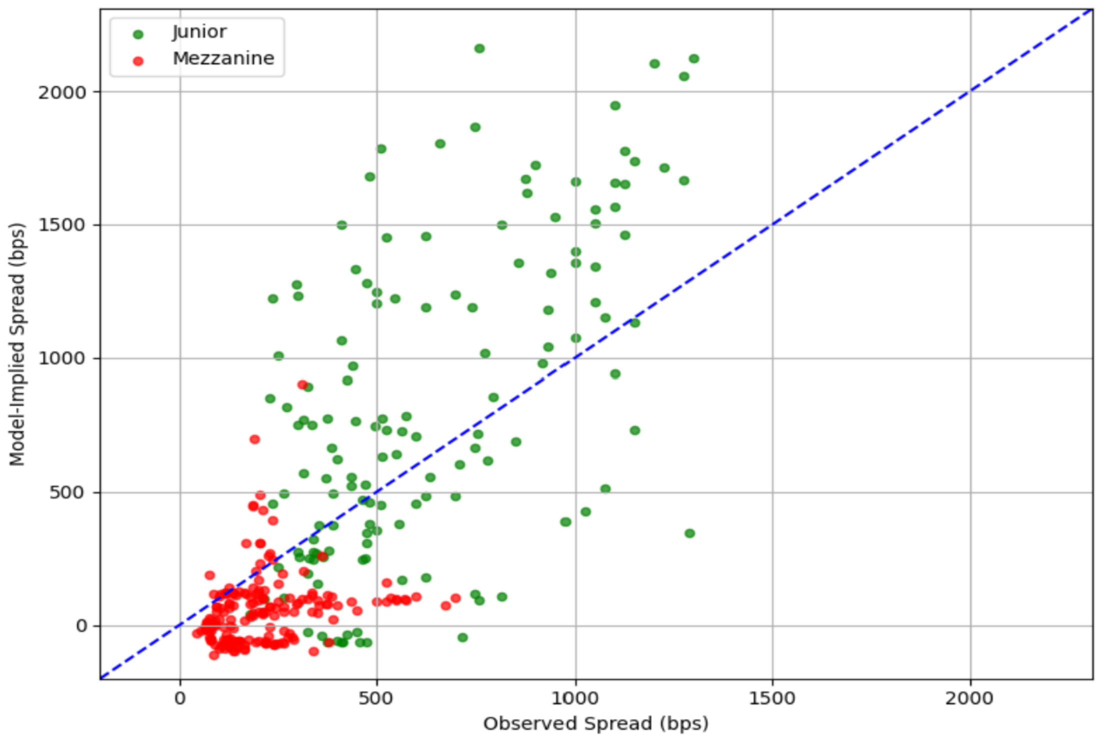
Our main exercise is to price the various CTR bonds issued by Freddie Mac between the inception of the CRT program in 2013 and the end of 2023. Table A.1 presents the complete list of tranches we price, indicating the tranche name, issuance date, attachment and detachment point, legal maturity, observed tranche spread, and model-implied tranche spread.

To price each tranche, we construct an initial distribution $(K_0^i, H_0^i, N_0^i, r_0^{m,i})$ by randomly sampling 1,000 households from the underlying loan portfolio that backs the specific deal, reported by Freddie Mac's Clarity Data Intelligence. We construct initial assets A_0^i as in (40). We draw 5,000 monthly sample paths of the aggregate shocks ϵ_t, and form sample paths for the aggregate state vector z_t from (1) starting from the observed initial state z_0. In each round, we draw idiosyncratic income and house price shocks for the 1,000 households. We also draw i.i.d. shocks for layoffs and loan resolution delay, as described above. The parameters of the VAR in (1), the market price of risk parameters in (7), as well as the prepayment parameters in (27) and (41) are estimated recursively: When pricing the CRT bonds issued in year $t+1$, we only use data up until December of year t to estimate the parameters. We solve for the CRT tranche spreads $ts^{[b,b']}$ from (37) averaging over the 5,000 sample paths to compute expectations. For each CRT bond/vintage we use the corresponding CE thresholds that trigger unscheduled principal paydowns.

Figure 11 summarizes our main results. It plots the observed tranche spreads (on the x-axis) against the model-implied spreads (on the y-axis), using different colors for the different types of tranches. Overall, our model delivers sensible credit spreads across tranches and vintages.

Comparing the model-implied and observed spreads across all 329 CRT tranches in our sample, we find that the model-implied CRT spread is on average 3.1 basis points lower than the observed

Figure 11: CRT Tranche Spreads: Model vs. Data



Notes: The figure shows the observed CTR tranches spreads at origination in the data for all 329 STACR tranches issued between the start of 2013 and 2025 on the x-axis and the corresponding model-implied tranche spreads $ts^{[b,b']}$ on the y-axis. Tranche spreads are expressed in basis points in annual yield terms. Junior tranches are those with attachment points lower or equal to 1%.

spreads. That is, the GSEs have paid CRT investors fairly, according to our model.

We study the relationship between model-implied and observed spreads in Table 3. The dependent variable in each column is the difference between the spread in the model and the spread in the data. Column 1 includes an indicator variable for junior tranches ($Junior = 1$), where we define a junior tranche to be any tranche with an attachment point lower or equal to 1%.[22] The coefficient on *Junior* is 341.2 basis points, indicating that the model implies much higher spreads for the riskiest tranches than what investors actually received for taking on the first-loss risk. In contrast, the model indicates -149.3 basis points lower spread than what was actually paid on the mezzanine CRT tranches. The average fair pricing hides interesting heterogeneity across types of tranches. According to our model, the GSEs did not compensate CRT investors nearly enough for the riskiest bonds, but over-compensated them for the safer mezzanine tranches. From the perspective of a CRT investor who takes compensation for interest rate risk, corporate credit risk, and housing market risk as given, the mezzanine CRT bonds are cheap (the spreads are too high, the prices too low) whereas the junior tranches are expensive (the spreads are too low). Figure 11

[22] All B, B-1, and B-2 tranches are junior tranches, but so are some M-3 and M-2 tranches in earlier vintages.

confirms the relative pricing of junior (red dots, mostly below the 45-degree line) and mezzanine tranches (green dots, mostly above the 45-degree line).

Column 2 of Table 3 incorporates the role of loan-to-value. The *High-LTV* dummy is one for all tranches that belong to a HA- or HQA-series deal, which refer to collateral pools of loans with LTV ratios above 80 percent. The coefficient on the *High-LTV* dummy and the *High-LTV* × *Junior* dummy are 53.6 and 382.7, and measured precisely. The mezzanine tranches of high-LTV deals are less underpriced than in low-LTV deals; the junior tranches of high-LTV deals are much more overpriced. Spreads on junior bonds in high-LTV deals are 565 bps too low, whereas spreads on junior bonds in low-LTV deals are "only" 182 bps too low.

Table 3: Model-Minus-Observed CRT Spreads

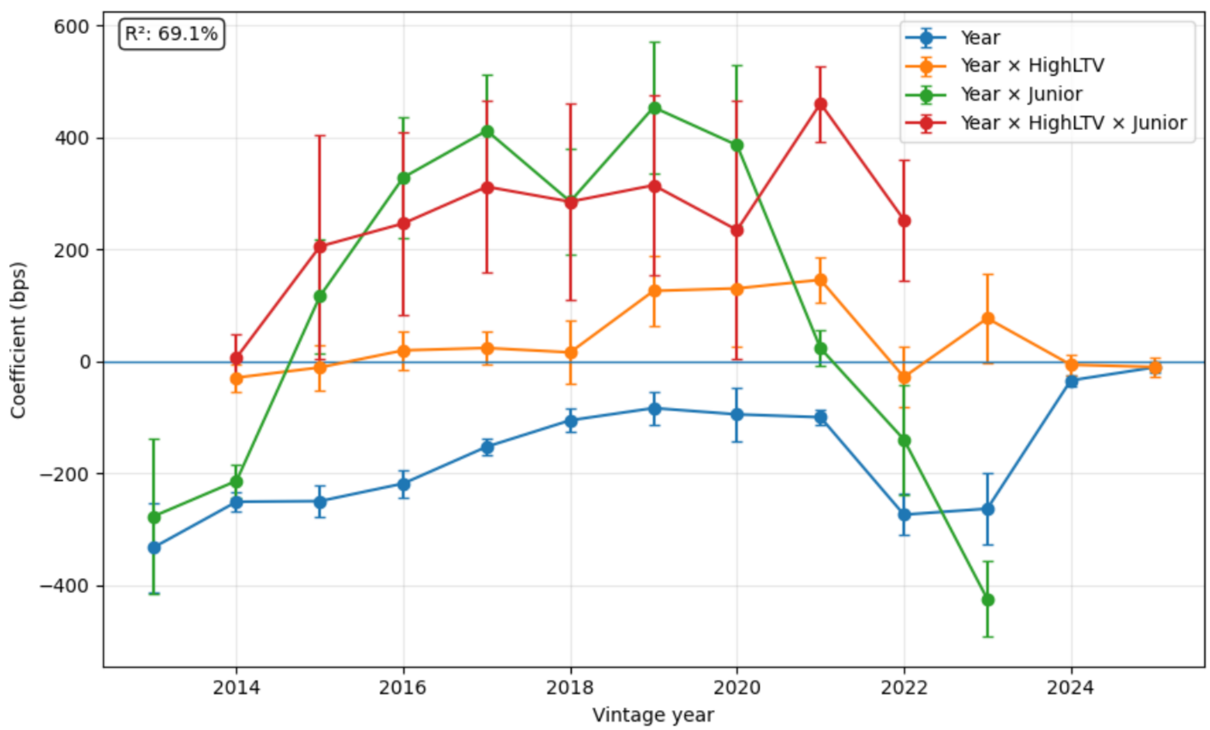
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-149.3***	-173.0***	-178.9***	-122.4***	-119.0***	-197.4***
Junior	341.2***	182.3***	90.0**	-319.9***	-310.0***	-362.0***
High LTV		53.6**	73.8***	8.4	8.2	11.7
High LTV × *Junior*		382.7***	340.6***	9.4	10.2	25.0
R^2 (%)	20.1	34.4	27.4	36.9	35.2	46.4
Observations	329	329	329	329	329	329
Average (bps)	-3.1	-3.1	-47.4	-254.1	-246.5	-343.0

Notes: The table reports regression models with the model-implied credit spread minus the observed credit spread as dependent variable. Column (1) contains an intercept and a *Junior* tranche dummy. Column (2) adds a *High LTV* deal dummy and the interaction of the *High LTV* deal and *Junior* tranche dummies. Column (1) and (2) are for the main model, estimated recursively. Column (3) is for the model estimated on the full sample. Columns (4)-(6) are for the recursive model model without housing premia, without housing and corporate credit risk premia, and without any risk premia, respectively. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Next, we explore how the pricing gap varies by origination year. We visualize vintage-specific effects from our full specification that includes Year fixed effects and interactions with high LTV and Junior dummies in Figure 12. The specification attains a high in-sample fit ($R^2 = 69.1\%$), indicating that basic vintage, deal, and seniority characteristics go a long way towards accounting for the cross-sectional variation in CRT mispricing. The blue line shows that mezzanine bonds in low-LTV deals paid spreads that were too high in each of the years; the difference between the model and the data is consistently negative. This gap shrinks for high-LTV deals as indicated by the positive coefficient plotted in the orange line. Adding up the blue and orange lines, mezzanine bonds in high-LTV are underpriced, except for the 2019–21 vintages. Table A.2 shows the standard errors on the vintage effects for junior bonds.

The green line shows the overpricing of junior bonds in low-LTV deals for all vintages from 2015 to 2021. We attribute the initial generosity of the 2013–15 junior spreads to the cost of establishing a new market. Model uncertainty on the part of investors, maybe combined with the recent memories of the GFC, led CRT investors to demand and receive high spreads to take on what turned out to be minimal credit risk. After this initial "learning phase," we see that the excess compensation for taking on credit risk begins to fall in 2016, reversing sign for junior bonds. The red line shows a positive and significant marginal effect for junior bonds in high-LTV deals. As we analyze below, high risk premia in corporate credit and housing markets translate into substantial risk premia in the CRT market. Junior bonds in high-LTV deals are much more exposed to this credit risk than mezzanine bonds in low-LTV deals. One potential explanation of the relative mispricing is that market participants underestimate the differential pricing of credit risk in junior versus mezzanine bonds and in high- versus low-LTV deals.

Figure 12: Gap Between Model-Implied and Observed CRT Spreads

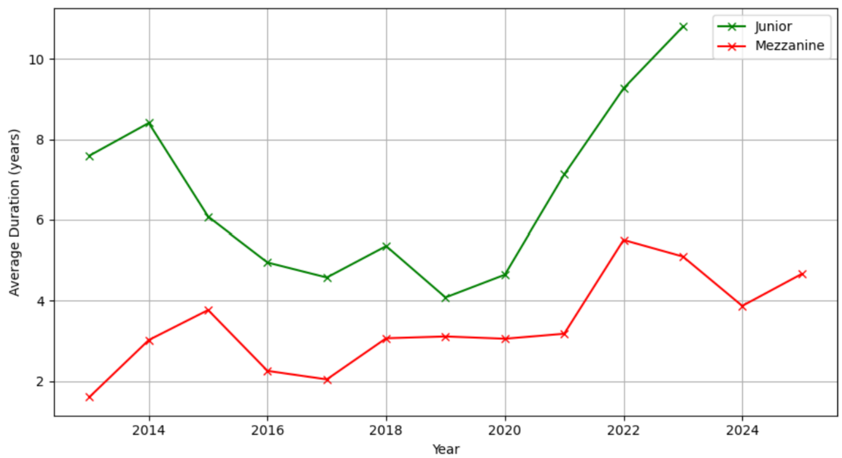


Notes: The figure plots the coefficients from a regression that has the difference between the model-implied and observed credit spread on each CRT bond, expressd in basis points, as its dependent variable and a series of fixed effects as its independent variables: vintage year (Year), Year × *High-LTV*, Year × *Junior*, and Year × *Junior* × *High-LTV*. The dots show the point estimates and the whiskers indicate ±1 standard error. There are no junior and/or high-LTV bonds in the last few vintages. The regression model's R^2 is 69.1%.

Prepayment risk is another potential driver of the relative mispricing of junior and mezzanine tranches. The mezzanine tranches not only take on less credit risk exposure than the junior tranches, they also take it on for less time. Figure 13 shows the average duration for junior and mezzanine tranches by vintage in the model. Junior bonds have an average duration of 5.87 years while mezzanine bonds have a duration of 3.70 years. There is time-variation in this duration gap as well. The market may not properly account for the cross-sectional difference in duration between junior and mezzanine tranches, as well as for the time variation in this difference.[23]

[23]The higher duration of the mezzanine tranches of the 2023, 2024, and 2025 tranches, which feature higher

Figure 13: Duration of CRT Bonds



Notes: The figure plots the McCauley duration of STACR tranches (expressed in years). At each origination date, we separately average the durations of the Junior and Mezzanine tranches. Junior tranches are those with attachment points lower or equal to 1%.

Finally, we note that the GSEs stopped issuing junior tranches in mid-2023. The last two deals with B tranches in the first half of 2023 paid a high credit spread to CRT investors, indeed a higher spread than what the model suggests. The model predicts substantial prepayment and hence a short duration for the 2023 vintage, given its high mortgage rates. CRT investors may not have appreciated the impact that the high mortgage rates of 2023 could have on shortening the duration of credit risk exposure of the vintage 2023-B1 bonds. The GSEs may have correctly concluded not to issue junior bonds if CRT investors demanded more spread than warranted. However, if the reason for the policy change were instead that the GSEs incorrectly perceived the high spreads on junior bonds to always have been too large (for example, relative to *realized* losses over the past decade), then the no-issuance decision would be harder to defend. The model suggests that the GSEs (the taxpayers) have been able to shed junior credit risk to the private sector on attractive terms since 2015.

5.2.1 The Role of Risk Pricing

To better understand the role of risk premia in the pricing of CRT tranches, we compute several special cases of the model. We relegate the detailed results to the appendix and discuss the main take-aways here.

First, we estimate the parameters of the VAR, prepayment function, and market prices of risk on the full sample and use those estimates to price all CRT bonds. Column 3 of Table 3 shows the results. This model generates an average gap of -47.4 basis points across all bonds, compared

mortgage rates, arises despite the fact that these vintages have higher predicted principal prepayments going forward. Instead the higher duration results from lower predicted credit loss than earlier vintages. It is also affected by the introduction of the A-1 bond in these vintages which we include among the mezzanine bonds, and which has a relatively high duration itself, and which lengthens the duration of the mezzanine tranches in the same deal.

to -3.1 basis points for the benchmark model with recursive estimation. It produces a similar gap between model and data spreads for mezzanine bonds but a smaller spread for junior bonds. Table A.2 shows that the difference in the model spreads between the recursive and the full-sample is most pronounced for the junior tranches issued between 2015 and 2019 and nearly vanishes thereafter. The means of house price appreciation and aggregate income growth rate under the $\mathbb{Q}$ measure are substantially lower in the recursive estimation in the 2015–17 period than the full-sample estimates.[24] This meaningfully increases default risk and hence junior CRT spreads in the main model compared to the full-sample estimation. This difference in pricing underscores the importance of using information that was actually available to market participants in real time.

In a second exercise, we shut down the housing risk premium. We zero out market prices of risk associated with the aggregate house price growth shock, the sixth row Λ_t in (7). We revert to the benchmark model's assumption of estimating VAR, prepayment, and MPR coefficients recursively. Column 4 of Table 3 shows the results. This model predicts lower spreads for all tranches, with average spreads that are 254.1 basis points lower than in the data. The large positive gap between model and data for junior tranche spreads found in the benchmark model disappears and reverses sign. Switching off housing risk premia lowers the model-implied spread on junior bonds by 502 bps in low-LTV deals and by 875 bps in high-LTV deals.

House price growth under the risk-neutral measure is substantially negative, while it is substantially positive under the physical measure. When the market price of house price risk is zeroed out, the risk-neutral house price dynamics is much closer to the physical dynamics. Since one of the conditions for default is for the homeowner to be underwater on the mortgage under the risk-neutral measure, defaults are much lower in the model where house price risk is not priced. In other words, the high model-implied credit spreads of the junior CRT tranches in the benchmark model crucially depend on the estimated market price of aggregate house price risk.

We also ask a related question: How much large does the housing risk premium need to be for the model to match the average CRT spreads on the mezzanine (junior) tranches? We find that increasing (decreasing) nry from 0.045/12 in the benchmark model to 0.065/12 (0.022/12) results in junior tranche spreads that match the data.[25]

Third, we also shut down the pricing of corporate credit risk, zeroing out both the third and sixth rows of Λ_t. Interest rate risk is the only remaining priced risk. Column 5 of Table 3 shows that this produces similar credit spreads as Column 4 (-246.5 bps versus -254.1 bps).

Fourth, we shut down all risk prices, setting all rows of Λ_t to zero. This is a model of risk-neutral pricing. Column 6 of Table 3 shows the results. This model produces a 96.5 bps reduction in credit spreads overall. Compared to the model with interest rate risk, the model without interest rate risk features higher income growth and higher housing returns under $\mathcal{Q}$, and hence smaller credit spreads for both mezzanine and junior bonds.

[24]This reflects higher credit and housing risk premia in the 2015–17 recursive-model estimation than in the full-sample estimation. The high market prices of credit and house price risk result from average credit and housing returns despite below-average quantities of macro risks in 2015–17.

[25]A higher nry leads the model to choose a lower value for the market price of housing risk Λ_0^h. The corresponding average monthly housing return risk premia is 0.792% (0.434%) compared to our model-implied 0.625%.

6 Pricing the Overall Mortgage Guarantee

Having developed a model for pricing mortgage credit risk, we now price the risk in the entire mortgage pool. We compute the guarantee fee g^{fee} produced by the model for each year using (38). The g-fee is a credit spread on an entire vintage year of mortgages, expressed relative to the one-month discount factor. The simulation approach is identical to that used for the CRT tranche pricing, except that we sample 5,000 loans/households from the standard loan origination file. For each mortgage loan, we use the actual first payment date to initialize the state vector. The advantage of our approach, as opposed to approaches that rely on extrapolation from CRT prices, is twofold. First, it does not require an assumption on how the tranche pricing would change were the volume of CRT issuance to increase to cover more (or all) of the mortgage originations. Second, it avoids making arbitrary assumptions on how the price of catastrophic risk (the senior A-H bond retained by the GSEs) relates to the price of CRT bonds.

Prior to the GFC, the g-fee was around 20 basis points (bps) per year. Following the mortgage market meltdown and large loan losses on the pre-GFC vintages, there was consensus that the GSEs had underpriced the risk, with adverse consequences for financial stability (e.g., Elenev et al., 2016). From 2009 to 2012, the average g-fee increased from 22 to 36 bps, and from 2012 to 2013 it jumped from 36 to 51 bps (Federal Housing Finance Agency, 2014). Table 4 shows the evolution of the g-fee in the CRT era. The g-fee increased further from 51 bps in 2013 to 65 bps in 2024. The total g-fee consists of a credit and a non-credit component. The non-credit component is 20 bps (Freddie Mac, 2017), consisting of 8 bps for administering the GSEs, 2 bps for the cost of securitizing loans, and 10 bps of general taxes levied by the Temporary Payroll Tax Cut Continuation Act of 2011. Our model-implied g-fee should be compared to the credit-risk component. In the data, the latter increased from 31 to 45 bps between 2013 and 2024.

Our model-implied g-fee is reported in the third column of Table 4. Our results indicate that the fair compensation for mortgage credit risk mortgages is very similar to what the GSEs have been charging since the GFC. Weighted by origination volume, the average g-fee over these 12 years is 38.71 bps in the data versus 37.79 bps in the model. This 1 basis point average difference hides some interesting time-series variation. According to our model, the GSEs overcharged for credit risk in four of the twelve years anywhere between 1 bps to 10 bps. They undercharged in eight of the twelve years anywhere between 1 bps to 12 bps. In the last two years of the sample, observed g-fees exceeded their model-implied counterparts by nearly 10 bps.

6.1 Decomposing the Guarantee Fee

The fourth and fifth columns of Table 4 decompose the model-implied g-fee into an expected loss (EL) and a credit risk premium (CRP) component. The expected loss is the g-fee that would be set by a risk-neutral insurer, or equivalently the guarantee fee risk neutral investors would accept to take over all the credit risk from the GSEs. The credit risk premium is the extra compensation a risk-averse investor/insurer demands for the fact that mortgage losses go up in bad times. The model's CRP varies both because the quantity of risk fluctuates across vintages and because the

Table 4: Guarantee Fees

Vintage	Observed		Model-implied			
	Total	Credit	Credit	EL	CRP	Tail
2013	51.0	31.0	41.86	10.61	31.25	18.00
2014	58.0	38.0	39.03	11.90	27.13	14.79
2015	59.0	39.0	44.60	13.03	31.57	19.71
2016	56.0	36.0	48.14	9.90	38.24	22.90
2017	53.0	33.0	45.27	14.29	30.98	19.61
2018	55.0	35.0	42.22	17.60	24.62	16.62
2019	56.0	36.0	38.92	14.84	24.08	14.90
2020	54.0	34.0	26.98	4.82	22.16	7.48
2021	56.0	36.0	27.57	2.15	25.42	7.65
2022	61.0	41.0	43.41	6.47	36.94	20.45
2023	66.0	46.0	36.51	24.71	11.81	12.89
2024	65.0	45.0	36.29	16.74	19.56	11.77
Average	58.71	38.71	37.79	12.31	25.48	14.51

Notes: The first column reports the observed average g-fee by vintage, obtained from the Federal Housing Financing Agency's annual reports to the Congress. It is the sum of the ongoing fee, which is the same for all borrowers, and the average among borrowers of the annual equivalent of upfront loan level price adjustments. The second column reports the actual credit-risk component, computed as the observed g-fee from the first column minus 20 basis points for non-credit related charges. The third column is the model-implied g-fee, which pertains to the credit risk component. The fourth and fifth column break out the model-implied g-fee into an expected loss (EL) and a credit risk premium (CRP) component. The last column prices a senior tranche which absorbs losses above a 5% loss rate. The last row weights the costs by the UPB amount of Freddie Mac's entire guarantee portfolio, obtained from its 10K annual reports.

market prices of risk vary over time. The weighted average EL across vintages is 12.31 bps, but ranges from 2.15 bps in 2021 to 24.71 bps in 2023. The CRP is 25.48 bps on average, ranging from 38.24 in 2016 to 22.16 bps in 2020 to 36.94 bps in 2022. The CRP falls to an all-time low of 11.81 bps in 2023 and stays relatively low in 2024.

The low expected losses in 2020–22 are noteworthy, given that they occur when the CRP is still high. The main reason for the low EL and the high CRP in 2020–22 is very fast house price appreciation (HPA). On the one hand, high HPA lowers the probability of loss, conditional on default as the underlying collateral has appreciated strongly. This force lowers the EL. On the other hand, the housing risk premium is high in this period of significant macroeconomic risk, which induces a large gap between the expected house price growth under the $\mathbb{P}$ and $\mathbb{Q}$ measures. The latter results in a high CRP component.

The low CRP in 2023–24 drives the low model-implied g-fees in the last two years of the sample. The low CRP in these last two vintages results not only from a lower housing risk premium but also from higher predicted future prepayment rates, which reduce the duration of credit risk exposure of the recent vintages.

6.2 What to Charge for Tail Risk?

The last column reports the fair risk premium for bearing the catastrophic loss risk. For consistency across vintages, we define catastrophic losses to be all losses in excess of a 5% loss rate. This corresponds approximately to the risk that the GSEs hold on to in the form of the senior A-H tranche. This tail risk component is hard to infer from CRT prices, i.e., hard to compute without a complete default, prepayment, and interest rate risk model. We find that fair compensation for tail risk exposure is 14.51 bps on average across years (weighted by UPB), and ranges from 7.65 bps in 2021 to 22.90 bps in 2016. Hence, catastrophic loss risk exposure that is retained by the GSEs accounts for about one-third of the overall risk. This is a surprising result since losses rarely exceed 5%. However, the states of the world in which such large losses do occur are particularly painful, hence the large "disaster risk premium." In a future privatization of the GSEs where the government is ultimately on the hook for catastrophic loss risk, and may seek ex-ante compensation for this (explicit or implicit) guarantee, this calculation should be informative.

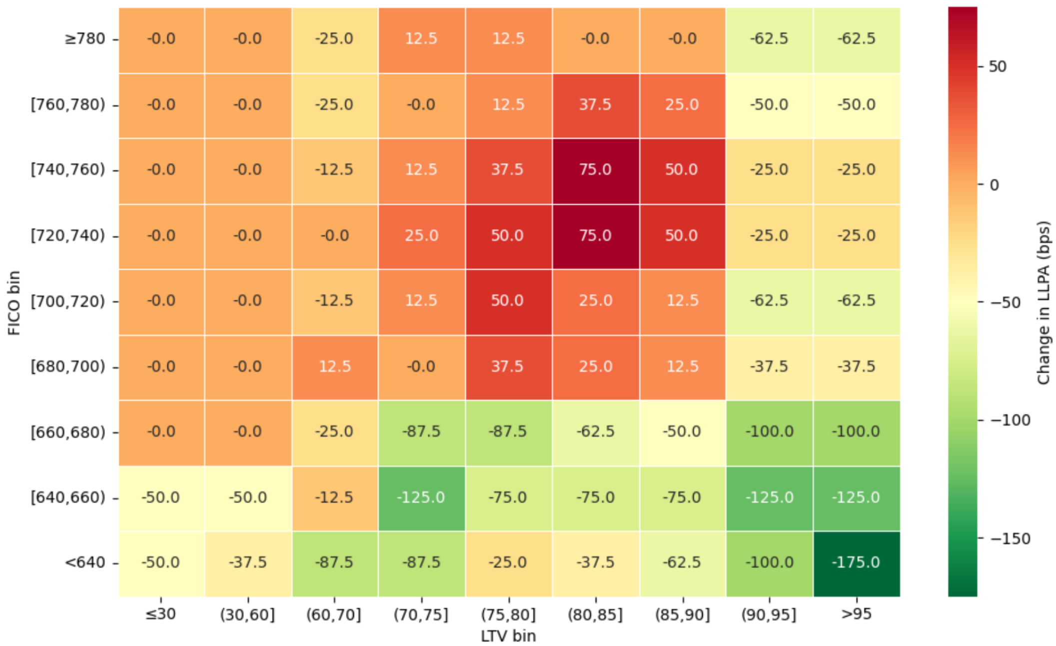
7 The Cross-Section of Mortgage Risk

Having developed a rich model of mortgage pricing, our last exercise uses the model to study the cross-section of mortgage credit risk.

Loan-level price adjustments (LLPAs) were introduced by the GSEs in April 2008 in response to the financial crisis. LLPAs are an upfront fee charged on conforming mortgages based on a borrower's risk profile. The goal of the LLPAs was to mitigate risk and increase capital for the GSEs. Between April 2008 and May 2023, LLPAs underwent several incremental adjustments.[26] A May 2023 LLPA reform was more substantial. Figure 14 shows which borrowers pay higher LLPAs after May 2023 in warmer colors (red), and those that pay lower LLPAs in cooler colors (green). Appendix Figure A.3 shows the current, post-reform LLPA grid. The reform lowered LLPAs for low-credit score borrowers, especially those with moderate to high LTV ratios, and increased fees for high-credit score, moderate-LTV borrowers (Federal Housing Finance Agency). It emphasized equity and affordability, drawing political controversy. We use our model to ask the positive question whether the reform brought the pricing closer to or farther from actuarially fair, risk-based pricing, leaving aside the normative question of how much redistribution through the LLPA grid is socially desirable.

[26] An adverse market delivery charge of 0.25% was introduced in late 2008 to reflect a period of heightened credit risk. It was lifted in 2015. Between 2009 and 2011, the GSEs introduced more detailed credit score and LTV buckets, higher pricing for lower-credit and high-LTV borrowers. They also introduced new LLPAs for investment properties, cash-out refinances, condominiums, and multiple-unit properties. LLPAs were increased substantially in 2013–2014, including hikes for middle-credit borrowers (FICO scores between 680 and 740) and for moderate LTV ranges (LTV between 75% and 85%). These increases were reversed in 2015. In January 2022, the FHFA increased LLPAs on high-balance loans and second homes.

Figure 14: Loan Level Price Adjustments: May 2023 Reform



Notes: The graph displays the change in Loan-Level Price Adjustments (LLPA) by credit score (FICO) and loan-to-value (LTV) group. Warm colors (orange/red) indicate higher fees under the new grid while cooler colors (yellow/green) indicate lower fees. Source: Federal Housing Finance Agency

7.1 LLPAs in the Model

To calculate LLPAs in the model, we sample 30,000 loans from the 2024 standard loan file. We group them into the same $(LTV, FICO)$ buckets that are used in the LLPA grid. The sample composition of the 2024 vintage is summarized in Table A.4. Because some buckets are sparsely populated in the data, for pricing accuracy purposes, we ensure that each bucket contains at least 1,000 loans by sampling with replacement when necessary.

We begin by solving for the actuarially fair credit spread for a specific $(FICO, LTV)$ group, $cs^{[F,L]}$ by applying the asset pricing formula (37):

$$\mathbb{E}^{\mathbb{Q}}\left[\sum_{s=1}^{T} \exp\left(-\sum_{t=1}^{s} \frac{y_t(1)}{12}\right)\left(\frac{cs^{(F,L)} + y_s(1)}{12} V_s^{[0,1]} + (F_s^{[0,1]} - F_{s-1}^{[0,1]})\right)\right] = 1, \tag{42}$$

where the terminal value is zero because the mortgages have fully amortized by time T. We simulate 5,000 sample paths of the state variables to compute the expectation.[27]

Next, we subtract the model-implied g-fee, which represents the UPB weighted-average spread

[27] We use the same 5,000 draws of the state vector in each vintage and for each (FICO, LTV) group. The draws are the same as those we used in the pricing of CRT tranches and to obtain g-fees.

among all $(FICO, LTV)$ groups. Denote the relative credit spread by

$$g^{(F,L)} = cs^{(F,L)} - g^{fee}.$$

Finally, we turn the relative credit spread $g^{(F,L)}$ from an annual credit spread into an upfront amount, expressed as a share of the origination UPB. The LLPA is the present discounted value of the credit spread over the life of the loan:

$$LLPA^{(F,L)} = \frac{\sum_{i\in(F,L)} \sum_{s=1}^{T} \exp\left(-\sum_{t=1}^{s} \frac{y_t(1)}{12}\right) \frac{g^{(F,L)}}{12} N_s^i}{\sum_{i\in(F,L)} N_0^i}, \tag{43}$$

where N_s^i denotes the outstanding balance at time s for loan i. To compute how this balance evolves, we consider both regular principal amortization and a fixed prepayment rate of 1.5% per month.[28] Our approach generates $g^{(F,L)} < 0$ and hence $LLPA^{(F,L)} < 0$ for buckets whose credit risk is below the vintage (UPB-weighted) average, and vice versa for above-average credit risk buckets.

Figure 15 shows model-implied LLPAs for the 2024 vintage. Warm colors (red cells) denote $(FICO, LTV)$ groups who have higher than average credit risk under actuarially fair pricing, denoted by positive numbers. Cells in green denote low-risk borrowers whose LLPAs are below average, denoted by negative numbers. LLPAs generally increase when reading from the top-left to the bottom-right of the graph. [29] Our model implies a large spread whereby the riskiest borrowers should pay upward of 2.2% in LLPAs and the safest borrowers should receive more than 1% in LLPAs. Holding fixed the LTV in the (80%,85%] range, the model-implied LLPA spread between borrowers with FICO scores below 640 and those with FICO scores above 780 is 215 bps. This is close but slightly smaller than the 250 bps spread in the post-2023 reform LLPA grid (reported in Table A.3). The slope of the credit spread surface in the FICO dimension is slightly steeper in the data than in the model for most of the LTV distribution.

7.2 Assessing the 2023 LLPA Reform

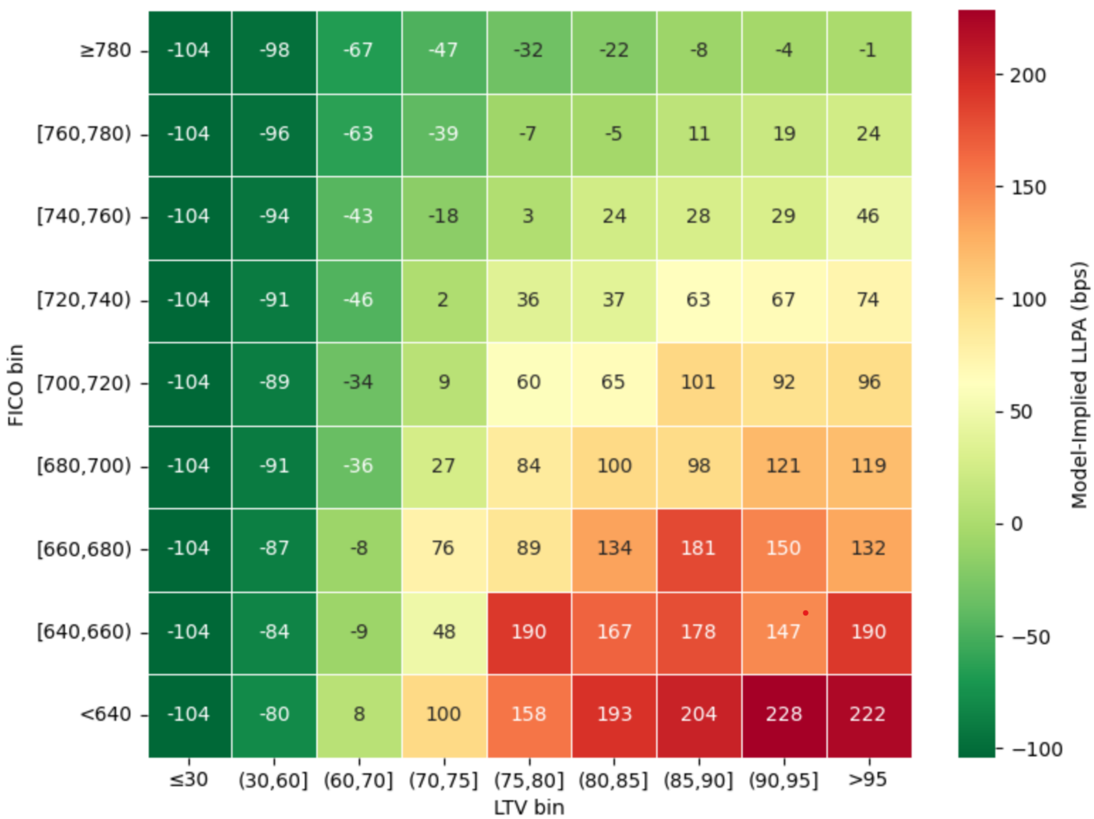
Finally, we ask whether the May 2023 reform brought the LLPA grid closer to the actuarially fair grid. To put model and data on the same footing, we first rescale the LLPAs in the data so that they average to zero on an UPB-weighted average basis.[30] We demean LLPAs in the data

[28] We could use our prepayment model to come up with a vintage-specific prepayment rate that differs for each (FICO, LTV) group. However, this may obscure the pricing of credit risk. In reality, the LLPAs do not change when mortgage interest rate levels increase, which in the model and in the data, increases the risk of prepayment. Our assumption of a 1.5% SMM corresponds to a CPR of 16.6%, close to the historical average annualized prepayment rate.

[29] The careful reader may notice some non-monotonicities in the grid of 15. these arise from (i) the presence of PMI that kicks in and absorbs some of the losses above 80% LTV but not below, (ii) heterogeneous prepayment rates across groups, (iii) heterogeneous debt-to-income ratios across groups, and (iv) heterogeneity in mortgage interest rates.

[30] The LLPAs in the data are weakly positive for each bucket, and hence strictly positive on average. The baseline g-fee pertains to the lowest-risk group. For example, for the 2024 vintage, the baseline g-fee was 28 basis points (48 basis points minus 20 basis points for the non-credit risk related components) and the average LLPA added 17 basis points when expressed as an annual spread, for an average g-fee of 45 basis points charged for credit risk in 2024.

Figure 15: Model-Implied Loan-Level Price Adjustments



Notes: The graph shows model-implied loan-level price adjustments. As explained in the text, these are computed from annual credit spreads in each (FICO,LTV) group relative to the UPB-weighted average credit spread among all loans. These credit spreads are then converted into an upfront fee, expressed as a percent of UPB at origination. These model-implied LLPAs average to zero across the entire matrix, when weighted by the UPB in each cell.

separately for the 2023 (pre-reform) LLPA grid and for the (post-reform) 2024 grid, using the 2023 UPB by group and 2024 UPB by group for the respective weights. We then compute the UPB-weighted root mean squared error (RMSE) between the demeaned LLPAs in model and data across $(FICO, LTV)$ groups g:

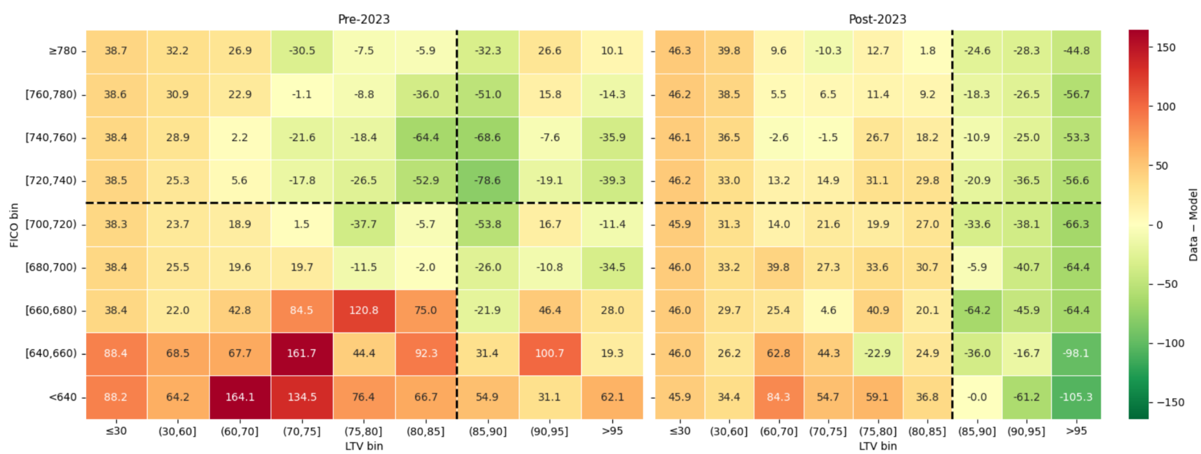
$$RMSE_t = \left(\sum_g w_t^g \left(LLPA_t^{g,data} - LLPA_t^{g,model} \right)^2 \right)^{0.5}$$

Note how this calculation is agnostic as to whether the average charge for credit risk is too high or too low. It is a purely cross-sectional calculation of the $(FICO, LTV)$ credit risk surface. We find an RMSE of 33.53 basis points for 2023 and of 29.28 basis points for 2024. This implies that the reform of May 2023 brought the cross-sectional pricing of mortgage credit risk somewhat closer to

Table A.3 shows how the g-fees and LLPAs are structured in the data.

the fair risk-based pricing implied by our model. This modest overall improvement in risk-based pricing hides larger differences across the LLPA matrix. We separately compute the RMSE for each of four quadrants of the LLPA matrix, splitting at a FICO score of 720 and a LTV ratio of 85%, and re-normalizing UPB weights within each quadrant. The 2023 reform reduced the RMSE the most in the low-FICO/low-LTV block, from 56.9 bps to 33.0 bps. It reduced it modestly in the high-FICO/high-LTV block, from 34.8 bps to 30.9 bps, and in the high-FICO/low-LTV borrowers, from 23.8 to 22.7 bps. In contrast, the RMSE increased meaningfully for low-FICO/high-LTV borrowers, from 34.1 to 47.5 bps, driving that group farther away from risk-based pricing.

Figure 16: Relative Risk Pricing: Data vs. Model Pre- vs. Post-Reform



Notes: The left panel plots the difference between the LLPAs in the Pre-Reform 2023 data and the LLPAs in the model, while the right panel plots the difference between the LLPAs in the Post-Reform 2024 data and the LLPAs in the (same) model. LLPAs in the data are demeaned by the UPB-weighted average of the LLPAs across all $(FICO, LTV)$ groups where the weights are based on the UPBs in the 2023 vintage in the left panel and the 2024 vintage in the right panel. Warm (red) colors indicate that LLPAs are higher in data than in model, the risk is over-priced, while cool colors (green) indicate that the risk is under-priced.

Figure 16 shows the difference between LLPAs in the pre-reform data and the model in the left panel and the same difference post-reform in the right panel. Red colors indicate that relative risk is priced higher in the data than in the model; those households are overcharged by the GSEs according to our model. Green colors indicate households for whom relative risk is underpriced in the data. The figure shows that the spread on low-FICO borrowers was too high pre-reform. The reform substantially reduced the overpricing of low-FICO borrowers. For those with LTV ratios below 85%, the spreads remain elevated post-reform. In contrast, credit spreads went from being too high to being too low for borrowers with a LTV ratio in excess of 85%.[31] For example, the riskiest group of borrowers on the grid with FICO score below 640 and LTV above 95% went from

[31] Subtracting the numbers in the right panel from the numbers in the left panel of Figure 16, cell by cell, does not exactly reproduce the changes in LLPAs brought about by the reform. The latter were reported in Figure 14. The reason is that we separately demean the observed LLPA grid in 2023 and 2024. Because the mean LLPA shifted substantially from 61.4 bps to 53.8 bps, this demeaning does not preserve the change in the level of LLPA. Figure 16 focuses on the change in relative pricing.

being 62 bps overcharged to 105 bps undercharged.

In sum, while the reform correct some of the cross-subsidization from low-FICO to high-FICO borrowers, it introduced a new issue of subsidizing high-LTV borrowers. The new LLPA matrix implicitly encourages leverage, which is potentially dangerous for financial stability.[32]

An open question is how a potential privatization of the GSEs would affect the cross-sectional pricing of risk across the household distribution. If a privatization resulted in actuarially fair pricing throughout, low-LTV–and among them especially low-FICO– households would win out at the expense of borrowers with a LTV ratio above 85%.

7.3 Can the Model Generate A Steeper FICO and Flatter LTV Slope?

Our risk-based pricing model generates a substantial slope in LLPAs across the LTV distribution. Holding fixed the FICO bin at $[720, 740)$, the model generates a slope in credit spreads between the highest and lowest LTV bins of 181 bps. This slope exceeds the corresponding empirical slope of 125 bps post-reform. Along the FICO dimension, the model generates a slope that is smaller than in the data for the borrowers with loan-to-value ratios below 85%. Holding fixed the LTV bin at $(70, 75]$, the model-implied spread between the lowest- and highest-FICO bins is 147 bps compared to a 212 bps spread in the data.

Is the model capable of generating a 212 (125) bps spread for the FICO (LTV) dimension under different assumptions? Yes. Appendix A.7 shows different ways of generating the observed FICO and LTV slopes in LLPAs in the model. To increase the slope in credit spreads along the FICO dimension, the first way is to crank up the sensitivity of credit risk premium to the credit spread, governed by $\Lambda_1^{cr,cs}$. This results in a lower drift of aggregate income under $\mathbb{Q}$, which in turn generates more income-driven defaults. Low-FICO borrowers are particularly sensitive to this risk. However, the resulting credit risk premium in this economy is highly counter-factual. The second way is to modify the mapping from FICO scores to initial asset levels, given by (40) in the benchmark calibration. By giving low-FICO borrowers a lot fewer initial assets, we can increase their relative exposure to aggregate income risk enough to reproduce the observed FICO slope in LLPAs. However, this model generates default rates that are too high relative to the historical sample. In sum, the data discipline the model's parameters in ways that make the high FICO slope in LLPAs in the data hard to reconcile with the model.

To lower the slope in credit spreads along the LTV dimension, we can lower the net rental yield and hence the housing risk premium. This results in less mortgage default risk and smaller differences between high- and low-LTV borrowers. However, the net rental yield becomes counter-factually low compared to the previous estimation.

How come that Kim et al. (2024) finds that the FICO slope in observed LLPAs is not steep

[32] How this reform affects welfare is beyond the scope of this paper but would depend on the welfare weights the social planner accords to each credit-risk group. Most models of incomplete risk sharing would have the social planner prescribe a subsidy for high marginal-utility (presumably low-FICO, high-LTV) borrowers, paid for by low marginal-utility (high-FICO, low-LTV) borrowers. Richer models would also consider the negative externalities from high leverage and mortgage defaults (e.g., Elenev et al., 2016).

enough, while we find that it is slightly too steep? The difference in results is driven by three key different modeling assumptions. First, our asset pricing model features a stochastic discount factor driven by aggregate credit, interest rate, and housing risk factors that affect the pricing of LLPAs, which is–as usual–done under the risk-neutral measure. Their model features a constant discount rate and they price LLPAs under the physical measure. Second, their pricing exercise applies the historical default probabilities by FICO-LTV bins to the current period. Our model generates the same default rates as in the data *along the historical path of state variables*. However, this path which included the GFC was quite unusual. The model implies lower default probabilities along most sample paths, and hence on average, going forward. Third, they assume a single loan in each FICO-LTV bucket, which amounts to an assumption of perfect correlation of default risk across loans within each bucket. Our pricing approach considers 1,000 loans in each bucket and recognizes that there is substantial scope for diversification of idiosyncratic risk across the loans within a bucket. Indeed, if we assume that there is a single loan in each bucket, our model also generates a much larger FICO slope.

8 Conclusion

This paper provides the most comprehensive asset pricing analysis to date of mortgage credit risk after the Great Financial Crisis. The post-GFC era saw the introduction of a large new asset class, the Credit Risk Transfer bonds. These instruments have allowed the Government Sponsored Enterprises, Freddie Mac and Fannie Mae, to lay off residential mortgage credit risk to private investors. We analyze all 12 years of the CRT market's pricing history. Contrary to received wisdom, which often confuses ex-post with ex-ante default risk, we find that the GSEs have paid the CRT investors compensation for taking over the credit risk that was fair, consistent with the pricing of risk in treasury and corporate bond as well as housing markets. This fair pricing hides interesting variation across vintages and between junior and mezzanine CRT bonds. Junior tranche spreads were mostly too low, while mezzanine spreads were uniformly too high. One potential explanation suggested by our model is the differential duration of credit risk exposure between the tranches due to prepayment risk.

We use the model to compute the fair guarantee fee, the cost of insuring all conforming mortgages in the U.S. The g-fee has increased substantially in the data post-GFC. We find that the GSEs have charged a g-fee that on average has been fair. About one-third of the model-implied g-fee is compensation for catastrophic losses, losses the GSEs do not transfer to the private sector. This g-fee calculation is relevant in the context of proposals to re-privatize the GSEs that preserve the government guarantee on agency-backed mortgage-backed securities.

Finally, we use our model to study the relative pricing of credit risk in the cross-section of households. We find evidence for substantial cross-subsidization flowing from low-FICO to high-FICO borrowers before the reform. The loan-level price adjustment reform of May 2023 reduced this cross-subsidization. However, the reform introduced new transfers from low-LTV to high-LTV borrowers, potentially encouraging more leverage in housing markets.

References

V. Acharya, S. Kon, S. Oncu, M. Richardson, S. Van Nieuwerburgh, and L. White. The government sponsored enterprises. In V. Acharya, T. Cooley, M. Richardson, and I. Walter, editors, *Regulating Wall Street*. John Wiley and Sons, Sept. 2010.

V. Acharya, M. Richardson, S. Van Nieuwerburgh, and L. White. *Guaranteed to Fail: Freddie, Fannie, and the Debacle of U.S. Mortgage Finance*. Princeton University Press, Mar. 2011.

V. Acharya, M. Richardson, S. Van Nieuwerburgh, and L. White. Guaranteed to fail: Fannie mae and freddie mac and what to do about them. *Economist Voice*, 10(1):15–19, 2013.

T. Adrian, E. Etula, and T. Muir. Financial intermediaries and the cross-section of asset returns. *Journal of Finance*, 69(6):2557–2596, 2014.

A. Ang and M. Piazzesi. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics*, 50(4):745–787, 2003.

N. Boyarchenko, A. Fuster, and D. O. Lucca. Understanding mortgage spreads. *Review of Financial Studies*, 32(10):3799–3850, October 2019. doi: 10.1093/rfs/hhz004. URL `https://doi.org/10.1093/rfs/hhz004`.

J. Y. Campbell and J. F. Cocco. A model of mortgage default. *Journal of Finance*, 70(4):1495–1554, 2015.

J. Y. Campbell, S. Giglio, and P. Pathak. Forced sales and house prices. *American Economic Review*, 101(5):2108–2131, August 2011. doi: 10.1257/aer.101.5.2108.

A. Capponi, R. Jia, and D. Rios. The effect of mortgage forbearance on refinancing: Evidence from the covid-19 period. Technical Report 68, 2021.

A. Carriero, T. E. Clark, and M. Marcellino. Vars, common factors, and the empirical implications of covid-19. *The Review of Economics and Statistics*, 103(5):1009–1024, 2021. doi: 10.1162/rest_a_00958.

M. Chernov, B. R. Dunn, and F. A. Longstaff. Macroeconomic-driven prepayment risk and the valuation of mortgage-backed securities. *Review of Financial Studies*, 31(3):1132–1183, 2018.

M. Davis and S. Van Nieuwerburgh. Housing, finance, and the macro-economy. In G. Duranton, J. V. Henderson, and W. C. Strange, editors, *Handbook of Regional and Urban Economics*, chapter 12, pages 735–811. Elsevier, 2015.

Y. Demyanyk and O. Van Hemert. Understanding the subprime mortgage crisis. *Review of Financial Studies*, 24(6):1848–1880, 2011.

Y. Deng, J. M. Quigley, and R. Van Order. Mortgage terminations, heterogeneity, and the exercise of mortgage options. *Econometrica*, 68(2):275–307, 2000.

P. Diep, A. L. Eisfeldt, and S. Richardson. The cross section of mbs returns. *Journal of Finance*, 76(5):2093–2151, October 2021.

A. Eisfeldt and A. Demers. Rental yields and hpa: The returns to single family rentals. NBER Working Paper 21804, National Bureau of Economic Research, Cambridge, MA, 12 2015. URL http://www.nber.org/papers/w21804.

V. Elenev, T. Landvoigt, and S. Van Nieuwerburgh. Phasing out the gses. *Journal of Monetary Economics*, 81:111–132, Aug. 2016.

J. Favilukis, S. C. Ludvigson, and S. Van Nieuwerburgh. The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium. *Journal of Political Economy*, 125(1):140–223, 2017. doi: 10.1086/689606. URL https://doi.org/10.1086/689606.

Federal Housing Finance Agency. FHFA Announces Updates to the Enterprises' Single-Family Pricing Framework. https://www.fhfa.gov/news/news-release/fhfa-announces-updates-to-the-enterprises-single-family-pricing-framework. News Release.

Federal Housing Finance Agency. Fannie mae and freddie mac single-family guarantee fees in 2013. Technical report, Federal Housing Finance Agency, 2014. URL https://www.fhfa.gov/reports/single-family-guarantee-fees/2013. Accessed: 2025-07-13.

Federal Housing Finance Agency. Performance of fannie mae's and freddie mac's single-family credit risk transfer. Technical report, Federal Housing Finance Agency (FHFA), May 2021. URL https://www.fhfa.gov/document/performance-of-fannie-maes-and-freddie-macs-single-family-credit-risk-transfer. FHFA Research Paper.

D. Finkelstein, A. Strzodka, and J. Vickery. Credit risk transfer and de facto gse reform. *Economic Policy Review*, 24(3):88–116, 2018.

T. Flanagan. Do gses subsidize mortgage lending? Working Paper 2025-03-011, Fisher College of Business, 2025.

W. Frame, A. Fuster, J. Tracy, and J. Vickery. The rescue of fannie mae and freddie mac. *Journal of Economic Perspectives*, 29(2):25—52, 2015.

Freddie Mac. What credit risk transfer tells us about guarantee fees: A single-family white paper. White paper, Freddie Mac, July 2017. URL https://www.freddiemac.com/fmac-resources/perspectives/pdf/CRT-GFee-White-Paper-Final.pdf.

A. Fuster and J. Vickery. Securitization and the fixed-rate mortgage. *Review of Financial Studies*, 28(1):176–211, 2015.

A. Fuster and P. S. Willen. Payment size, negative equity, and mortgage default. *American Economic Journal: Economic Policy*, 9(4):167–191, November 2017. doi: 10.1257/pol.20150237. URL https://www.aeaweb.org/articles?id=10.1257/pol.20150237.

A. Fuster, S. H. Lo, and P. S. Willen. The time-varying price of financial intermediation in the mortgage market. *The Journal of Finance*, 79(4):2553–2602, 2024. doi: 10.1111/jofi.13358. URL https://onlinelibrary.wiley.com/doi/10.1111/jofi.13358.

X. Gabaix, A. Krishnamurthy, and O. Vigneron. Limits of arbitrage: Theory and evidence from the mortgage-backed securities market. *Journal of Finance*, 62(2):557–595, 2007.

P. Ganong and P. Noel. Liquidity versus wealth in household debt obligations: Evidence from housing policy in the great recession. *American Economic Review*, 110(10):3100–3138, 2020a. doi: 10.1257/aer.20181243. URL https://doi.org/10.1257/aer.20181243.

P. Ganong and P. J. Noel. Why do borrowers default on mortgages? NBER Working Paper 27585, National Bureau of Economic Research, July 2020b. URL https://www.nber.org/papers/w27585. Revised August 2022.

K. Gerardi, A. Lehnert, S. M. Sherlund, and P. S. Willen. Making sense of the subprime crisis. *Brookings Papers on Economic Activity*, 2008(2):69–159, 2008.

K. Gerardi, E. Rosenblatt, P. Willen, and V. Yao. Foreclosure externalities: New evidence. Technical Report Working Paper No. 15-24, Federal Reserve Bank of Boston, 2015. URL https://www.bostonfed.org/publications/research-department-working-paper/2015/foreclosure-externalities-new-evidence.aspx.

K. S. Gerardi, K. F. Herkenhoff, L. E. Ohanian, and P. S. Willen. Can't pay or won't pay? unemployment, negative equity, and strategic default. *The Review of Financial Studies*, 31(3):1098–1139, 2018. doi: 10.1093/rfs/hhx124.

P. Gete, A. Pavlov, A. Tsouderou, and S. Wachter. Forbearance and the cost of credit. Technical report, 2025. URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5241320#.

E. Golding and D. Lucas. Credit risk transfer and the pricing of mortgage default risk. Working paper, MIT Golub Center for Finance and Policy, October 2020. URL https://mitsloan.mit.edu/shared/ods/documents?PublicationDocumentID=9586. Preliminary draft analyzing CRT bond pricing using Vista Data Services.

A. Gupta and S. Van Nieuwerburgh. Valuing private equity investments strip by strip. *Journal of Finance*, 76(5):2009–2050, 2021.

D. Gupta. Too much skin-in-the-game?: The effect of mortgage market concentration on credit and house prices. *Review of Financial Studies*, 35(2):814–865, 2022. doi: 10.1093/rfs/hhab027.

F. Guvenen, F. Karahan, S. Ozkan, and J. Song. Lifetime incomes in the united states over six decades. *American Economic Review*, 109(5):1488–1541, 2019. doi: 10.1257/aer.20181586.

Z. He, B. Kelly, and A. Manela. Intermediary asset pricing: New evidence from many asset classes. *Journal of Financial Economics*, 126(1):1–35, 2017.

Y. Kim, F. Liu, and D. Zhang. The distributional effects of gse pricing on home purchases. Technical report, 2024. URL `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5222560`.

R. S. J. Koijen, H. K. Lee, and S. Van Nieuwerburgh. Aggregate lapsation risk. *Journal of Financial Economics*, 155:103819, 2024.

M. Lenza and G. E. Primiceri. How to estimate a var after march 2020. *Journal of Monetary Economics*, 135:1–19, 2022. doi: 10.1016/j.jmoneco.2022.03.005.

F. A. Longstaff and A. Rajan. An empirical analysis of the pricing of collateralized debt obligations. *The Journal of Finance*, 63(2):529–563, 2008. doi: 10.1111/j.1540-6261.2008.01330.x. URL `https://doi.org/10.1111/j.1540-6261.2008.01330.x`.

H. Lustig, S. Van Nieuwerburgh, and A. Verdelhan. The wealth-consumption ratio. *Review of Asset Pricing Studies*, 3(1):38–94, 2013.

D. McGowan and H. Nguyen. To securitize or to price credit risk? *Journal of Financial and Quantitative Analysis*, 58(1):289–323, Feb. 2023.

R. C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29(2):449–470, 1974. doi: 10.2307/2978814. URL `https://doi.org/10.2307/2978814`.

A. Mian and A. Sufi. The consequences of mortgage credit expansion: Evidence from the u.s. mortgage default crisis. *Quarterly Journal of Economics*, 124(4):1449–1496, 2009. doi: 10.1162/qjec.2009.124.4.1449. URL `https://doi.org/10.1162/qjec.2009.124.4.1449`.

J. E. O'Neill. The market view of mortgage credit risk. Working paper, UCLA Anderson School of Management, June 2022. URL `https://ssrn.com/abstract=4152078`. Builds a pricing model using TRACE and hand-collected CRT issuance data; analyzes implied guaranty-fees and portfolio credit risk.

G. E. Primiceri and A. Tambalotti. Macroeconomic forecasting in the time of covid-19. *Brookings Papers on Economic Activity*, 2020(Summer):1–50, 2020.

F. Schorfheide and D. Song. Real-time forecast evaluation with covid-19 data. *Journal of Econometrics*, 220(1):3–22, 2021. doi: 10.1016/j.jeconom.2020.07.037.

T. M. Sinai and J. Heathcote. House price moments in boom–bust cycles. Technical Report 18059, National Bureau of Economic Research, Cambridge, MA, Jan. 2012. URL `https://www.nber.org/papers/w18059`.

K. J. Singleton. The structure of risks in equilibrium affine models of bond yields. *Journal of Finance*, 57(3):1307–1341, 2002.

A Appendix

A.1 Data: VAR elements

The variable r_t is the market yield on U.S. Treasury Securities at 1-Year Constant Maturity, Quoted on an Investment Basis (GS1), retrieved from FRED. The variable ys_t is constructed as the difference in ten-year and the one-year treasury yields using Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis GS10 from FRED. The variable cr_t is constructed as the monthly log change in the ICE BofA BBB US Corporate Total Return Index Value (BAMLCC0A4BBBTRIV), received from FRED. We use Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity (BAA10Y) retrieved from FRED for s_t. We use the monthly log change in the S&P CoreLogic Case-Shiller U.S. National Home Price Index (CSUSHPINSA), retrieved from FRED, to construct the house price growth rate, h_t. To construct the aggregate disposable income growth rate, k_t, we use seasonally-adjusted disposable personal income per capita in current dollars (A229RC0), retrieved from FRED. For l_t, we use the series seasonally-adjusted total non-farm layoffs and discharges (JTSLDR), retrieved from FRED.

A.2 Realized Loss Calculations

The realized loss l_t^i on mortgage loan i is the positive difference between the default costs and default credits:

$$\begin{aligned}
&\text{Realized Loss} = \max\{\text{Default Costs} - \text{Default Credits}, 0\} \\
&\text{Default Costs} = \text{Default UPB} + \text{Accrued Interest} + \text{T \& I} + \text{Legal Fees} + \text{Maintenance Fees} \\
&\text{Default Credits} = \text{Net Sales Proceeds} + \text{PMI} + \text{Miscellaneous Credits}
\end{aligned}$$

We define each of these components in turn.

The two main default costs are the UPB that remains outstanding at the time of loss resolution and the accrued interest on the defaulted UPB between the time of default and the time of loss resolution. These components are computed from the model as explained below. For the remaining three cost components, we apply the averages reported in Freddie Mac's CRT Data Intelligence report. Taxes and insurance (T&I) are on average 4.2% of the defaulted UPB, legal fees 1.41%, and maintenance expenses 1.99%. Balanced against these default costs are three credits. The first one is the net proceeds from selling the collateral. The second one is credit from private mortgage insurance (PMI). Both are discussed below. The third one is miscellaneous credits, which we assume equal 0.83% of the delinquent UPB, the observed average according to the same CRT Data Intelligence Report.

If borrower i defaults at time t, the amount of default UPB is

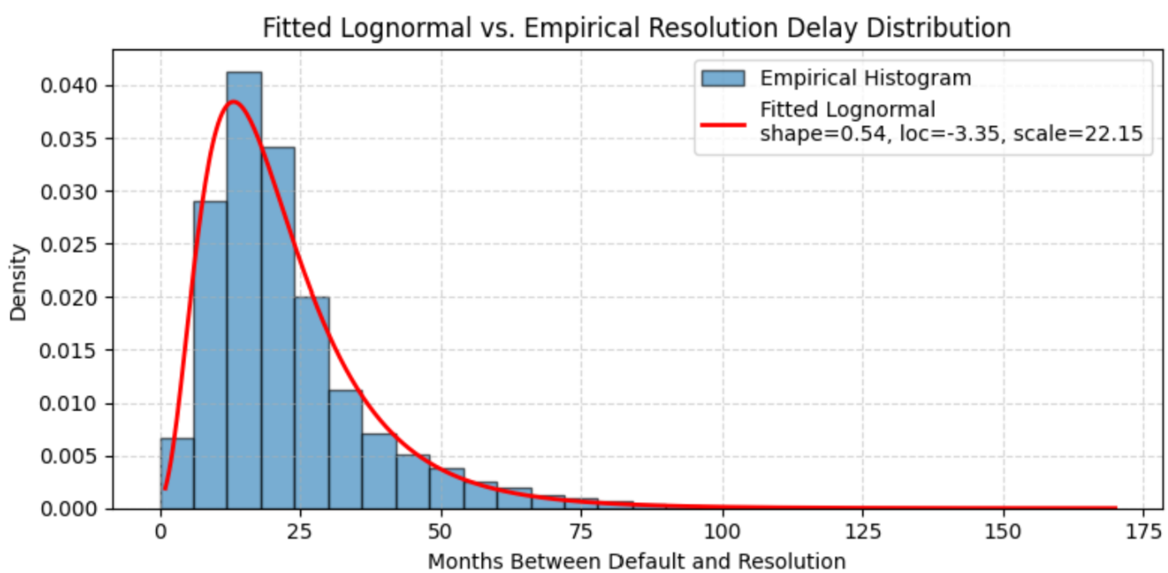
$$\text{Default UPB}_t^i := N_t^i \mathbf{1}_{\tau_t^i=1, \epsilon_t^i=0}$$

The accrued interest is the amount of missed interest rate payments between the first missed payment and final loan resolution on the default UPB. The loan resolution time of a borrower i who defaults at $t_{i,def}$ is: $t_{i,res} = t_{i,def} + \omega^i$. It follows that:

$$\text{Accrued Interest}_t^i = \omega^i \cdot \frac{r_0^{m,i}}{12} \cdot N_t^i.$$

Empirically, the time between default and resolution, ω_i, varies across time and space, and indeed across borrowers. Figure A.1 shows that a log-normal distribution fits the observed delay in the 2000-2024 vintages well. The resulting mean delay is 22 months, the median 18 months, and the distribution has a long tail extending out to 170 months. Based on this evidence, the resolution delay in the model ω^i is drawn from an i.i.d. lognormal distribution X for which $\log(X - \ell) \sim \mathcal{N}(\ln(e^{\mu}), \sigma)$. The best fit between model and data results in a shape parameter $\sigma = 0.54$, location parameter $\ell = -3.35$, and scale parameter $e^{\mu} = 22.15$.

Figure A.1: Distribution of Mortgage Default Resolution Delays



Notes: The histogram shows the time lag between mortgage default and mortgage loan resolution in the data for the 2000-2024 vintages in our sample. The red line shows the fitted log-normal curve we assume in our pricing model.

We denote the net proceeds from the sale of the collateral that back loans that defaulted at t as:

$$\text{Net Sales Proceeds}_t^i = R_{t_{i,res}}^i := (1 - fd_{t_{i,res}})(1 - sc)H_{t_{i,res}}^i$$

where fd^i is given by (39).

GSE mortgages with original LTV above 80% must take out private mortgage insurance (PMI). When loans with PMI coverage default, the mortgage insurer covers (some or all of) the loss. This is relevant for the HAQ series of CRT bonds, which contain loans with origination LTV above 80%.

It is also relevant for the LLPA discussion below. PMI coverage depends on the LTV ratio. For each LTV group v, the PMI covers a fixed fraction of the default cost:

$$PMI_t^i = \xi_v \cdot \text{Default Cost}_t^i.$$

For standard 30-year fixed-rate mortgages, the PMI coverage rate ξ_v is 12% for LTVs at origination in the (80%,85%] range, 25% for LTVs in the (85%,90%] range, 30% for LTVs in the (90%,95%] range, and 35% for LTVs in (95%, 97%]. PMI coverage automatically terminates when a borrower's LTV falls below 78% of the original home value. At that point, the CRT investor no longer enjoys the additional protection from PMI.[33] We note that the presence of PMI could make it such that the credit risk born by CRT investors in high-LTV pools (the HQA series) is actually lower than that born by CRT investors in low-LTV pools (the DNA series).

A.3 Mortgage Losses: Modified vs Non-modified Loans

We used the 2006 vintage to investigate whether mortgage loan modification significantly affects mortgage loss rates. Figure A.2 breaks out cumulative loss rates for the 2006 vintage by modification status: whether the loan was ever modified during its life. It shows that only one in six dollars of losses on this pool came from modified loans. In the first 10 years of the life of these loans, which is the relevant horizon for CRT investors, a much smaller percentage of losses came from modified loans.

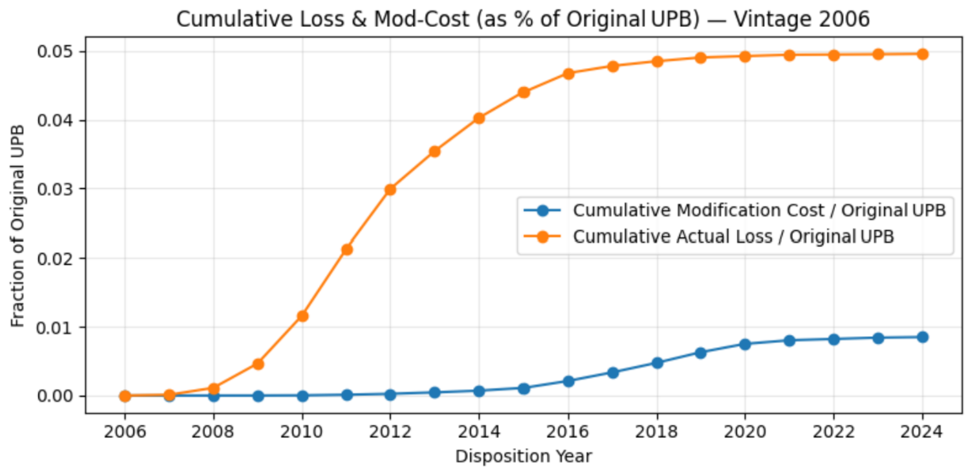
A.4 Principal Payments to CRT Investors

This appendix discusses the detailed rules that determine how scheduled and unscheduled principal payments on the reference pool flow through to the cash flows paid to CRT investors.

We begin by defining some terminology. We recall that the senior tranche is the A-H tranche, and is never sold to CRT investors. The subordinate tranches are those sold to CRT investors that are junior to (take credit losses before) the senior tranche. Define the Senior (Subordinate) Reduction Amount as the amount by which the principal of the senior (subordinate) tranche(s) is (are) paid down at each payment date. Define the Senior (Subordinate) Percentage as the ratio of the total outstanding senior (subordinate) tranche notional over the total outstanding notional of the entire pool. The credit enhancement (CE) is one minus the Senior Percentage, which is equal to the Subordinate Percentage. Additionally, there is Offered Reference Tranche Percentage (ORTP) at each payment date, which is defined as the percentage of the notional of the sold tranches at

[33] By the Homeowners Protection Act (HOPA) §4902(b), the servicer must terminate the PMI payment with no action by the borrower if the loan falls below 78% of the original value and the loan is current. Article §4902(c) also states that PMI terminates automatically at the midpoint of the loan, i.e., after 15 years for a 30-year mortgage. This is not relevant for us since it is beyond the horizon of CRT bonds. Paragraph §4902(a) of HOPA states that borrowers may also initiate PMI termination when the loan falls below 80% of the original home value, and that the servicer must oblige if the borrower is current, has a good payment history, and the property value has not declined. We do not apply this last rule since few borrowers are aware of this option, and establishing whether the true market value of a property has increased or not is difficult in practice.

Figure A.2: Cumulative Loss Rate by Modification Status



Notes: The blue line shows the cost as a percentage of the original UPB due to modifications. The orange line shows the loss rate excluding modification-related losses.

each payment date t divided by the original notional of the pool at time 0. These percentages change as the deal ages.

Scheduled principal payments refer to regular amortization. The part of each month's fixed mortgage payment that constitutes principal repayment grows over time as the mortgage ages. There are two types of *unscheduled* principal payments. One is called the *recovery principal* and the other is called the *unscheduled stated principal.* The recovery principal is the amount of recoveries from realized losses. The unscheduled stated principal consists of prepayments, the most important source of principal payments for both subordinate tranches and the senior tranche.

Recovery Principal All recovery principal payments flow exclusively to the senior tranche. CRT investors never receive principal payments from loss recoveries.

Scheduled Principal Payments For scheduled principal payments, there are two regimes. Prior to March 2018,[34] scheduled principal payments are allocated to the senior (A-H) tranche and the subordinate tranches pro rata according to the current-period Senior and Subordinate Percentages. Among the subordinate tranches, scheduled principal payments are allocated sequentially, starting with the most senior tranche.

After March 2018, the pro-rata allocation of scheduled principal payments to senior and subordinate STACR tranches only occurs when tests (1)–(3) below are satisfied. If any of tests (1), (2), or (3) fails, there is no scheduled principal payment to the subordinate tranches.

[34] This refers to all deals from the first CRT deal, 2013 STACR DN1, to the 2018 STACR DNA1 deal issued in March 2018, according to the information in the private placement memoranda.

Unscheduled Stated Principal Payments STACR subordinate tranches only receive unscheduled stated principal payments if tests (1), (2), and (3) below are satisfied. Unscheduled stated principal payments from the pool are then split pro rata between senior and subordinate tranches according to the current Senior and Subordinate Percentages. Among the subordinate tranches, stated principal payments are sequential, i.e., in strict order of seniority. If any of the tests (1), (2), or (3) fail, then unscheduled stated principal payments, including prepayments, stop flowing to the subordinate tranches and get diverted to the senior tranche.

Freddie Mac currently administers four tests related to principal payments:

1. Cumulative Net Loss Test
2. Minimum Credit Enhancement Test
3. Delinquency Test
4. Supplemental Subordinate Redution Amount test (SSRA Test)

We describe these tests in detail below, and explain the special role of test (4). We obtain the actual test thresholds for each deal by manually collecting information from the STACR deal documents (private placement memoranda) and Freddie Mac's Data Intelligence.

To illustrate the mechanics, suppose that the current subordinate percentage is 4.6%, all tests are passed, and the M-1 tranche is the most senior subordinate tranche with positive notional balance. Suppose that total (scheduled and stated) principal payments in month t from the reference pool are \$10,000. Then the M-1 tranche holders receive a \$460 cash flow in month t, which reduces the notional outstanding of the M-1 tranche by the same amount. The M-2, B-1 and B-2 tranches receive no principal payments and their notionals remain unchanged.

We now describe the three tests in more detail. The first test verifies whether the current cumulative level of realized loss on the collateral pool (between origination and the current month), as a share of the collateral pool's notional at origination, exceeds a threshold. The loss threshold ranges from 0.1% to 3% percent across deals, and is reported in the deal documents. In addition, within a given deal, the loss threshold varies over the life of the deal. For example, for the 2025-HQA1 deal, the cumulative loss threshold is 0.1% percent at the beginning of the deal and increases gradually to 1.1% after ten years, an increase of 0.1% each year. For Fixed-Severity CRT deals (the 2013 and 2014 vintages), the threshold starts at 0.25% and increases by 0.25% annually, and cumulative net loss is defined as the cumulative 180-day plus delinquency rate.

The second test computes the current credit enhancement (CE), which is the Subordinate Percentage in month t. That threshold ranges from 2% to 7% across deals, as reported in the deal documents.

The CE at origination may or may not be below the threshold. If the CE threshold is not met initially, it usually is met at some point later in the life of the deal.

The third test compares the delinquency rate over the most recent six-month period to a threshold. The definition of the delinquency rate varies across deals. It may refer to the 60-day plus, 90-day plus, or 180-day plus delinquency. The delinquency level is calculated as:

$$\textit{Delinquency level} := \frac{\frac{1}{6}\sum_{j=0}^{5} \text{ Delinquent Principal Balance }_{t-j}}{(\text{ Subordinate Percentage }) \times (\text{UPB}) - \text{Actual Loss Amount}}.$$

The threshold delinquency level is typically 0.5%. The delinquency test is only administered after month six of the deal. This test is administered after March 2015.

Prior to June 2018, if any of the three test fails, then the investors of the subordinate tranches still receive scheduled principal payments, excluding prepayments. After June 2018, if any of the three test fails, then the investors of the subordinate tranches receive no principal payments at all.

The Supplemental Subordination Reduction Amount After 2018, Freddie Mac began to administer a Supplemental Subordinate Reduction Amount (SSRA) Test to determine whether the subordinate tranches qualify for additional principal amortization. The first CRT bond that receives supplemental subordinate reduction amount is the STACR 2019 DNA4 Deal. It states that, if the CE at time t is sufficiently high, then an additional payment, called the Supplemental Subordinate Reduction Amount, will be allocated to the subordinate tranches. The SSRA test does not interfere with the three tests related to principal payments. Typically, the CE threshold in the SSRA test is either 5.5% or 6.15%, depending on the deal. The first STACR deal that implements a 5.5% SSRA threshold is the 2021-DNA5 Deal. Prior deals all use SSRA equal to 6.15%.

The Supplemental Subordinate Reduction Amount is defined as the outstanding UPB of the reference pool at time t times the difference between the actual credit enhancement in month t, the ORTP, and the threshold credit enhancement in the SSRA test:

$$\text{SSRA}_t = \text{UPB}_t \times \max(\text{ ORTP }_t - \text{ SSRA Threshold}, 0).$$

The GSEs collect the amount $ORTP_0$ times UPB_0 upfront from the CRT investors, and put it in an escrow account. They distribute some of the cash from that account, namely the amount $SSRA_t$ in month t, entirely to the subordinate tranches for those deals that satisfy test 4 in month t. Among the subordinate tranches, the SSRA is paid sequentially. To receive the SSRA, the three aforementioned tests do not need to be passed.

The A-1 Tranche Starting in November 2023, STACRs began to issue a new A-1 tranche, which is pari passu with the A-H tranche, which the GSEs retain. In other words, the A-1 bond is not a subordinate tranche but a senior tranche. In the first 36 months of its life, the scheduled and stated principal payments to the A-1 tranche are the same as for the A-H tranche. From month 36, all senior reduction amount, excluding the recovery principal, is allocated to the A-1 Tranche, regardless of whether the three tests are passed or not. In month 40, all remaining notional of the A-1 tranche is paid back as long as the cumulative net loss does not exceed 1% of the pool notional at that date.

A.5 Model-implied CRT tranche Spreads: Tranche by Tranche

We present the full CRT tranche spreads from 2013 to 2024 below. Recall that the interest rate benchmark was LIBOR for STACR deals issued between June 2013 (2013DN1) and Sept 2020 (2020 DNA4), and SOFR thereafter. Fixed severity bonds are labeled as the DN (low-LTV loans) and HQ series (high-LTV loans), while the actual loss deals are called DNA (low-LTV loans) and HQA (high-LTV loans) series.

Table A.1: STACR CRT Tranches: Model Implied vs Observed Spreads (%)

Deal	Tranche	Mat	Attach	Detach	Min CE	Obs Sprd	Mod Sprd
2013-DN1	M-1	10.00	1.95	3.00	3.00	3.40	-0.97
2013-DN1	M-2	10.00	0.30	1.95	3.00	7.15	-0.43
2013-DN2	M-1	10.00	1.65	3.00	3.00	1.45	-0.83
2013-DN2	M-2	10.00	0.30	1.65	3.00	4.25	-0.35
2014-DN1	M-1	10.00	3.50	4.51	5.00	1.00	-0.86
2014-DN1	M-2	10.00	2.00	3.50	5.00	2.20	-0.67
2014-DN1	M-3	10.00	0.30	2.00	5.00	4.50	-0.23
2014-DN2	M-1	10.00	3.50	4.50	3.00	0.85	-1.10
2014-DN2	M-2	10.00	2.00	3.50	3.00	1.65	-0.71
2014-DN2	M-3	10.00	0.30	2.00	3.00	3.60	-0.40
2014-DN3	M-1	10.00	3.60	4.60	5.10	1.35	-0.98
2014-DN3	M-2	10.00	2.40	3.60	5.10	2.40	-0.73
2014-DN3	M-3	10.00	0.40	2.40	5.10	4.00	-0.61
2014-DN4	M-1	10.00	4.20	5.20	5.70	1.40	-0.98
2014-DN4	M-2	10.00	2.90	4.20	5.70	2.40	-0.75
2014-DN4	M-3	10.00	0.50	2.90	5.70	4.55	-0.64
2014-HQ1	M-1	10.00	4.10	6.50	7.00	1.65	-0.88
2014-HQ1	M-2	10.00	2.55	4.10	7.00	2.50	-0.67
2014-HQ1	M-3	10.00	0.75	2.55	7.00	4.10	-0.65
2014-HQ2	M-1	10.00	4.10	6.10	6.60	1.45	-0.80
2014-HQ2	M-2	10.00	2.25	4.10	6.60	2.20	-0.66
2014-HQ2	M-3	10.00	0.60	2.25	6.60	3.75	-0.64
2014-HQ3	M-1	10.00	4.75	6.50	7.00	1.65	-0.93
2014-HQ3	M-2	10.00	3.10	4.75	7.00	2.65	-0.70
2014-HQ3	M-3	10.00	0.85	3.10	7.00	4.75	-0.65
2015-DN1	M-1	10.00	3.50	4.50	5.00	1.25	-0.78
2015-DN1	M-2	10.00	2.50	3.50	5.00	2.40	-0.67
2015-DN1	M-3	10.00	1.00	2.50	5.00	4.15	-0.62
2015-DN1	B	10.00	0.00	1.00	5.00	11.50	7.30
2015-DNA1	M-1	12.50	3.25	4.25	4.75	0.90	-0.71
2015-DNA1	M-2	12.50	2.25	3.25	4.75	1.85	0.47
2015-DNA1	M-3	12.50	1.00	2.25	4.75	3.30	2.50
2015-DNA1	B	12.50	0.00	1.00	4.75	9.20	9.80
2015-DNA2	M-1	12.50	4.50	5.50	6.00	1.15	-0.89
2015-DNA2	M-2	12.50	2.50	4.50	6.00	2.60	-0.65
2015-DNA2	M-3	12.50	1.50	2.50	6.00	3.90	0.23

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Deal	Tranche	Mat	Attach	Detach	Min CE	Obs Sprd	Mod Sprd
2015-DNA2	B	12.50	0.00	1.50	6.00	7.55	7.16
2015-DNA3	M-1	12.50	4.85	5.85	6.35	1.35	-0.74
2015-DNA3	M-2	12.50	2.65	4.85	6.35	2.85	-0.45
2015-DNA3	M-3	12.50	1.00	2.65	6.35	4.70	2.52
2015-DNA3	B	12.50	0.00	1.00	6.35	9.35	11.82
2015-HQ1	M-1	10.00	4.75	6.50	7.00	1.05	-0.76
2015-HQ1	M-2	10.00	3.00	4.75	7.00	2.20	-0.65
2015-HQ1	M-3	10.00	1.50	3.00	7.00	3.80	-0.63
2015-HQ1	B	10.00	0.00	1.50	7.00	10.75	5.14
2015-HQ2	M-1	10.00	4.10	5.60	6.10	1.10	-0.74
2015-HQ2	M-2	10.00	2.25	4.10	6.10	1.95	-0.64
2015-HQ2	M-3	10.00	1.00	2.25	6.10	3.25	-0.24
2015-HQ2	B	10.00	0.00	1.00	6.10	7.95	8.57
2015-HQA1	M-1	12.50	4.95	5.95	6.45	1.25	-0.83
2015-HQA1	M-2	12.50	2.70	4.95	6.45	2.65	-0.22
2015-HQA1	M-3	12.50	1.00	2.70	6.45	4.70	5.29
2015-HQA1	B	12.50	0.00	1.00	6.45	8.80	16.20
2015-HQA2	M-1	12.50	5.40	6.40	6.90	1.15	-0.64
2015-HQA2	M-2	12.50	2.95	5.40	6.90	2.80	-0.30
2015-HQA2	M-3	12.50	1.00	2.95	6.90	4.80	4.63
2015-HQA2	B	12.50	0.00	1.00	6.90	10.50	15.57
2016-DNA1	M-1	12.50	3.95	5.00	5.50	1.45	-0.59
2016-DNA1	M-2	12.50	2.95	3.95	5.50	2.90	-0.52
2016-DNA1	M-3	12.50	1.00	2.95	5.50	5.55	3.78
2016-DNA1	B	12.50	0.00	1.00	5.50	10.00	14.02
2016-DNA2	M-1	12.50	4.15	5.00	5.50	1.25	-0.55
2016-DNA2	M-2	12.50	3.25	4.15	5.50	2.20	-0.58
2016-DNA2	M-3	12.50	1.00	3.25	5.50	4.65	2.49
2016-DNA2	B	12.50	0.00	1.00	5.50	10.50	13.40
2016-DNA3	M-1	12.50	4.00	5.00	5.50	1.10	-0.53
2016-DNA3	M-2	12.50	3.05	4.00	5.50	2.00	-0.57
2016-DNA3	M-3	12.50	1.00	3.05	5.50	5.00	3.54
2016-DNA3	B	12.50	0.00	1.00	5.50	11.25	14.62
2016-DNA4	M-1	12.50	4.00	5.00	5.50	0.80	-0.51
2016-DNA4	M-2	12.50	3.00	4.00	5.50	1.30	-0.56
2016-DNA4	M-3	12.50	1.00	3.00	5.50	3.80	2.82
2016-DNA4	B	12.50	0.00	1.00	5.50	8.60	13.58
2016-HQA1	M-1	12.50	4.40	5.50	6.00	1.75	-0.55
2016-HQA1	M-2	12.50	3.20	4.40	6.00	2.75	-0.45
2016-HQA1	M-3	12.50	1.00	3.20	6.00	6.35	5.56
2016-HQA1	B	12.50	0.00	1.00	6.00	12.75	16.68
2016-HQA2	M-1	12.50	4.50	5.50	6.00	1.20	-0.52
2016-HQA2	M-2	12.50	3.00	4.50	6.00	2.25	-0.39
2016-HQA2	M-3	12.50	1.00	3.00	6.00	5.15	6.32
2016-HQA2	B	12.50	0.00	1.00	6.00	11.50	17.39
2016-HQA3	M-1	12.50	4.20	5.50	6.00	0.80	-0.51

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Deal	Tranche	Mat	Attach	Detach	Min CE	Obs Sprd	Mod Sprd
2016-HQA3	M-2	12.50	2.85	4.20	6.00	1.35	-0.11
2016-HQA3	M-3	12.50	1.00	2.85	6.00	3.85	6.68
2016-HQA3	B	12.50	0.00	1.00	6.00	9.00	17.25
2016-HQA4	M-1	12.50	4.28	5.50	6.00	0.80	-0.58
2016-HQA4	M-2	12.50	3.05	4.28	6.00	1.30	-0.48
2016-HQA4	M-3	12.50	1.00	3.05	6.00	3.90	4.95
2016-HQA4	B	12.50	0.00	1.00	6.00	8.75	16.70
2017-DNA1	M-1	12.50	2.55	3.75	4.25	1.20	-0.61
2017-DNA1	M-2	12.50	1.00	2.55	4.25	3.25	1.96
2017-DNA1	B-1	12.50	0.50	1.00	4.25	4.95	7.49
2017-DNA1	B-2	12.50	0.00	0.50	4.25	10.00	16.62
2017-DNA2	M-1	12.50	2.30	3.50	4.00	1.20	-0.34
2017-DNA2	M-2	12.50	1.00	2.30	4.00	3.45	2.70
2017-DNA2	B-1	12.50	0.50	1.00	4.00	5.15	7.75
2017-DNA2	B-2	12.50	0.00	0.50	4.00	11.25	16.51
2017-DNA3	M-1	12.50	2.50	3.50	4.00	0.75	-0.46
2017-DNA3	M-2	12.50	1.00	2.50	4.00	2.50	2.20
2017-DNA3	B-1	12.50	0.50	1.00	4.00	4.45	7.66
2017-DNA3	B-2	12.50	0.00	0.50	4.00	11.00	16.57
2017-HQA1	M-1	12.50	3.25	4.25	4.75	1.20	-0.57
2017-HQA1	M-2	12.50	1.00	3.25	4.75	3.55	3.78
2017-HQA1	B-1	12.50	0.50	1.00	4.75	5.00	12.03
2017-HQA1	B-2	12.50	0.00	0.50	4.75	12.75	20.56
2017-HQA2	M-1	12.50	3.00	4.00	4.25	0.80	-0.48
2017-HQA2	M-2	12.50	1.00	3.00	4.25	2.65	4.93
2017-HQA2	B-1	12.50	0.50	1.00	4.25	4.75	12.79
2017-HQA2	B-2	12.50	0.00	0.50	4.25	12.00	21.06
2017-HQA3	M-1	12.50	3.70	4.50	4.75	0.55	-0.22
2017-HQA3	M-2	12.50	1.00	3.70	4.75	2.35	4.56
2017-HQA3	B-1	12.50	0.50	1.00	4.75	4.45	13.33
2017-HQA3	B-2	12.50	0.00	0.50	4.75	13.00	21.23
2018-DNA1	M-1	12.50	3.10	4.00	4.50	0.45	-0.28
2018-DNA1	M-2	12.50	1.00	3.10	4.50	1.80	0.42
2018-DNA1	B-1	12.50	0.50	1.00	4.50	3.15	5.70
2018-DNA1	B-2	12.50	0.00	0.50	4.50	9.50	15.31
2018-DNA2	M-1	30.00	2.50	3.50	3.75	0.80	-0.18
2018-DNA2	M-2	30.00	1.00	2.50	3.75	2.15	1.01
2018-DNA2	B-1	30.00	0.50	1.00	3.75	3.70	5.50
2018-DNA2	B-2	30.00	0.00	0.50	3.75	10.50	15.05
2018-DNA3	M-1	30.00	3.00	4.00	4.25	0.75	-0.03
2018-DNA3	M-2	30.00	1.10	3.00	4.25	2.10	0.40
2018-DNA3	B-1	30.00	0.60	1.10	4.25	3.90	3.77
2018-DNA3	B-2	30.00	0.10	0.60	4.25	7.75	10.18
2018-HQA1	M-1	30.00	3.20	4.00	4.25	0.70	-0.08
2018-HQA1	M-2	30.00	2.10	3.20	4.25	2.30	-0.07
2018-HQA1	B-1	30.00	0.50	2.10	4.25	4.35	5.58

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Deal	Tranche	Mat	Attach	Detach	Min CE	Obs Sprd	Mod Sprd
2018-HQA1	B-2	30.00	0.00	0.50	4.25	11.00	19.49
2018-HQA2	M-1	30.00	3.00	4.00	4.25	0.75	-0.04
2018-HQA2	M-2	30.00	1.15	3.00	4.25	2.30	2.70
2018-HQA2	B-1	30.00	0.65	1.15	4.25	4.25	9.17
2018-HQA2	B-2	30.00	0.10	0.65	4.25	11.00	15.69
2019-DNA1	M-1	30.00	3.00	4.25	4.50	0.90	0.01
2019-DNA1	M-2	30.00	1.10	3.00	4.50	2.65	0.52
2019-DNA1	B-1	30.00	0.60	1.10	4.50	4.65	4.73
2019-DNA1	B-2	30.00	0.10	0.60	4.50	10.75	11.54
2019-DNA2	M-1	30.00	3.50	4.25	4.50	0.80	0.24
2019-DNA2	M-2	30.00	1.10	3.50	4.50	2.45	0.64
2019-DNA2	B-1	30.00	0.60	1.10	4.50	4.35	5.21
2019-DNA2	B-2	30.00	0.10	0.60	4.50	10.50	12.12
2019-DNA3	M-1	30.00	3.25	4.25	4.50	0.73	-0.01
2019-DNA3	M-2	30.00	1.10	3.25	4.50	2.05	2.33
2019-DNA3	B-1	30.00	0.60	1.10	4.50	3.25	8.96
2019-DNA3	B-2	30.00	0.10	0.60	4.50	8.15	14.98
2019-DNA4	M-1	30.00	3.00	4.00	4.25	0.70	-0.18
2019-DNA4	M-2	30.00	1.10	3.00	4.25	1.95	2.03
2019-DNA4	B-1	30.00	0.60	1.10	4.25	2.70	8.19
2019-DNA4	B-2	30.00	0.10	0.60	4.25	6.25	14.57
2019-HQA1	M-1	30.00	3.60	4.50	4.75	0.90	0.19
2019-HQA1	M-2	30.00	1.50	3.60	4.75	2.35	2.45
2019-HQA1	B-1	30.00	0.60	1.50	4.75	4.40	9.69
2019-HQA1	B-2	30.00	0.10	0.60	4.75	12.25	17.14
2019-HQA2	M-1	30.00	3.50	4.50	4.75	0.70	0.11
2019-HQA2	M-2	30.00	1.50	3.50	4.75	2.05	3.09
2019-HQA2	B-1	30.00	0.60	1.50	4.75	4.10	10.68
2019-HQA2	B-2	30.00	0.10	0.60	4.75	11.25	17.74
2019-HQA3	M-1	30.00	3.50	4.50	4.75	0.75	-0.09
2019-HQA3	M-2	30.00	1.50	3.50	4.75	1.85	4.47
2019-HQA3	B-1	30.00	0.60	1.50	4.75	3.00	12.34
2019-HQA3	B-2	30.00	0.10	0.60	4.75	7.50	18.68
2019-HQA4	M-1	30.00	3.25	4.50	4.75	0.77	-0.22
2019-HQA4	M-2	30.00	1.15	3.25	4.75	2.05	4.89
2019-HQA4	B-1	30.00	0.60	1.15	4.75	2.95	12.75
2019-HQA4	B-2	30.00	0.10	0.60	4.75	6.60	18.06
2020-DNA1	M-1	30.00	2.75	3.75	4.00	0.70	0.11
2020-DNA1	M-2	30.00	1.10	2.75	4.00	1.70	3.07
2020-DNA1	B-1	30.00	0.60	1.10	4.00	2.30	8.53
2020-DNA1	B-2	30.00	0.10	0.60	4.00	5.25	14.50
2020-DNA2	M-1	30.00	2.50	3.75	4.00	0.75	0.30
2020-DNA2	M-2	30.00	1.10	2.50	4.00	1.85	4.53
2020-DNA2	B-1	30.00	0.60	1.10	4.00	2.50	10.08
2020-DNA2	B-2	30.00	0.10	0.60	4.00	4.80	16.79
2020-DNA3	M-1	30.00	3.00	4.00	4.50	1.50	-0.56

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Deal	Tranche	Mat	Attach	Detach	Min CE	Obs Sprd	Mod Sprd
2020-DNA3	M-2	30.00	1.75	3.00	4.50	3.00	0.82
2020-DNA3	B-1	30.00	0.75	1.75	4.50	5.10	4.53
2020-DNA3	B-2	30.00	0.25	0.75	4.50	9.35	10.42
2020-DNA4	M-1	30.00	3.00	4.00	4.50	1.50	-0.58
2020-DNA4	M-2	30.00	1.75	3.00	4.50	3.75	0.78
2020-DNA4	B-1	30.00	0.75	1.75	4.50	6.00	4.57
2020-DNA4	B-2	30.00	0.25	0.75	4.50	10.00	10.74
2020-DNA5	M-1	30.00	2.50	3.50	3.75	1.30	-0.02
2020-DNA5	M-2	30.00	1.50	2.50	3.75	2.80	1.05
2020-DNA5	B-1	30.00	0.75	1.50	3.75	4.80	3.82
2020-DNA5	B-2	30.00	0.10	0.75	3.75	11.50	11.34
2020-DNA6	M-1	30.00	2.00	3.00	3.25	0.90	-0.00
2020-DNA6	M-2	30.00	1.25	2.00	3.25	2.00	0.95
2020-DNA6	B-1	30.00	0.75	1.25	3.25	3.00	2.76
2020-DNA6	B-2	30.00	0.25	0.75	3.25	5.65	7.27
2020-HQA1	M-1	30.00	3.00	4.25	4.50	0.75	1.89
2020-HQA1	M-2	30.00	1.10	3.00	4.50	1.90	6.98
2020-HQA1	B-1	30.00	0.60	1.10	4.50	2.35	12.24
2020-HQA1	B-2	30.00	0.10	0.60	4.50	5.10	17.87
2020-HQA2	M-1	30.00	3.00	4.00	4.25	1.10	0.51
2020-HQA2	M-2	30.00	1.10	3.00	4.25	3.10	9.03
2020-HQA2	B-1	30.00	0.60	1.10	4.25	4.10	14.99
2020-HQA2	B-2	30.00	0.10	0.60	4.25	7.60	21.60
2020-HQA3	M-1	30.00	3.30	4.00	4.50	1.55	-0.58
2020-HQA3	M-2	30.00	1.75	3.30	4.50	3.60	2.60
2020-HQA3	B-1	30.00	0.75	1.75	4.50	5.75	7.87
2020-HQA3	B-2	30.00	0.25	0.75	4.50	10.00	13.55
2020-HQA4	M-1	30.00	3.00	4.00	4.25	1.30	-0.61
2020-HQA4	M-2	30.00	1.75	3.00	4.25	3.15	2.06
2020-HQA4	B-1	30.00	0.75	1.75	4.25	5.25	7.35
2020-HQA4	B-2	30.00	0.25	0.75	4.25	9.40	13.18
2020-HQA5	M-1	30.00	2.75	3.75	4.00	1.10	-0.04
2020-HQA5	M-2	30.00	1.50	2.75	4.00	2.60	1.96
2020-HQA5	B-1	30.00	0.75	1.50	4.00	4.00	6.22
2020-HQA5	B-2	30.00	0.25	0.75	4.00	7.40	11.92
2021-DNA1	M-1	30.00	2.00	2.50	2.75	0.65	-0.05
2021-DNA1	M-2	30.00	1.25	2.00	2.75	1.80	0.34
2021-DNA1	B-1	30.00	0.75	1.25	2.75	2.65	1.01
2021-DNA1	B-2	30.00	0.25	0.75	2.75	4.75	3.46
2021-DNA2	M-1	12.50	2.00	2.50	2.75	0.80	0.02
2021-DNA2	M-2	12.50	1.25	2.00	2.75	2.30	0.72
2021-DNA2	B-1	12.50	0.75	1.25	2.75	3.40	2.74
2021-DNA2	B-2	12.50	0.25	0.75	2.75	6.00	7.09
2021-DNA3	M-1	12.50	2.00	2.50	2.75	0.75	0.03
2021-DNA3	M-2	12.50	1.25	2.00	2.75	2.10	0.47
2021-DNA3	B-1	12.50	0.75	1.25	2.75	3.50	1.57

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Deal	Tranche	Mat	Attach	Detach	Min CE	Obs Sprd	Mod Sprd
2021-DNA3	B-2	12.50	0.25	0.75	2.75	6.25	4.86
2021-DNA5	M-1	12.50	1.75	2.00	2.00	0.65	-0.15
2021-DNA5	M-2	12.50	1.25	1.75	2.00	1.65	0.19
2021-DNA5	B-1	12.50	0.75	1.25	2.00	3.05	2.58
2021-DNA5	B-2	12.50	0.25	0.75	2.00	5.50	6.43
2021-DNA6	M-1	20.00	1.75	2.00	2.25	0.80	0.02
2021-DNA6	M-2	20.00	1.25	1.75	2.25	1.50	1.29
2021-DNA6	B-1	20.00	0.75	1.25	2.25	3.40	3.23
2021-DNA6	B-2	20.00	0.25	0.75	2.25	7.50	6.65
2021-DNA7	M-1	20.00	1.75	2.25	2.50	0.85	0.05
2021-DNA7	M-2	20.00	1.30	1.75	2.50	1.80	0.79
2021-DNA7	B-1	20.00	0.75	1.30	2.50	3.65	2.58
2021-DNA7	B-2	20.00	0.25	0.75	2.50	7.80	6.21
2021-HQA1	M-1	12.50	2.50	3.25	3.50	0.70	0.01
2021-HQA1	M-2	12.50	1.25	2.50	3.50	2.25	2.60
2021-HQA1	B-1	12.50	0.75	1.25	3.50	3.00	7.53
2021-HQA1	B-2	12.50	0.25	0.75	3.50	5.00	12.47
2021-HQA2	M-1	12.50	2.50	3.00	3.25	0.70	0.01
2021-HQA2	M-2	12.50	1.25	2.50	3.25	2.05	3.11
2021-HQA2	B-1	12.50	0.75	1.25	3.25	3.15	7.69
2021-HQA2	B-2	12.50	0.25	0.75	3.25	5.45	12.25
2021-HQA3	M-1	20.00	2.00	3.25	3.50	0.85	1.19
2021-HQA3	M-2	20.00	1.25	2.00	3.50	2.10	4.32
2021-HQA3	B-1	20.00	0.75	1.25	3.50	3.35	7.51
2021-HQA3	B-2	20.00	0.25	0.75	3.50	6.25	11.92
2021-HQA4	M-1	20.00	2.00	3.50	3.75	0.95	0.49
2021-HQA4	M-2	20.00	1.25	2.00	3.75	2.35	3.96
2021-HQA4	B-1	20.00	0.75	1.25	3.75	3.75	7.76
2021-HQA4	B-2	20.00	0.25	0.75	3.75	7.00	12.36
2022-DNA1	M-1A	20.00	3.00	4.50	4.75	1.00	0.21
2022-DNA1	M-1B	20.00	2.00	3.00	4.75	1.85	0.39
2022-DNA1	M-2	20.00	1.25	2.00	4.75	2.50	0.88
2022-DNA1	B-1	20.00	0.75	1.25	4.75	3.40	2.47
2022-DNA1	B-2	20.00	0.25	0.75	4.75	7.10	6.06
2022-DNA2	M-1A	20.00	3.40	4.75	4.75	1.30	0.21
2022-DNA2	M-1B	20.00	2.00	3.40	4.75	2.40	0.38
2022-DNA2	M-2	20.00	1.25	2.00	4.75	3.75	1.13
2022-DNA2	B-1	20.00	0.75	1.25	4.75	4.75	3.08
2022-DNA2	B-2	20.00	0.25	0.75	4.75	8.50	6.88
2022-DNA3	M-1A	20.00	3.60	5.25	5.25	2.00	0.49
2022-DNA3	M-1B	20.00	2.00	3.60	5.25	2.90	0.52
2022-DNA3	M-2	20.00	1.25	2.00	5.25	4.35	0.92
2022-DNA3	B-1	20.00	0.75	1.25	5.25	5.65	1.68
2022-DNA3	B-2	20.00	0.25	0.75	5.25	9.75	3.89
2022-DNA4	M-1A	20.00	3.60	5.25	5.25	2.20	0.51
2022-DNA4	M-1B	20.00	2.00	3.60	5.25	3.35	0.53

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Deal	Tranche	Mat	Attach	Detach	Min CE	Obs Sprd	Mod Sprd
2022-DNA4	M-2	20.00	1.25	2.00	5.25	5.25	0.90
2022-DNA4	B-1	20.00	0.75	1.25	5.25	6.25	1.79
2022-DNA4	B-2	20.00	0.25	0.75	5.25	10.25	4.26
2022-DNA5	M-1A	20.00	3.60	5.25	5.25	2.95	0.82
2022-DNA5	M-1B	20.00	2.05	3.60	5.25	4.50	0.57
2022-DNA5	M-2	20.00	1.25	2.05	5.25	6.75	0.74
2022-DNA5	B-1	20.00	0.75	1.25	5.25	7.50	1.19
2022-DNA5	B-2	20.00	0.25	0.75	5.25	12.90	3.48
2022-DNA6	M-1A	20.00	3.60	4.75	4.75	2.15	1.32
2022-DNA6	M-1B	20.00	2.20	3.60	4.75	3.70	0.80
2022-DNA6	M-2	20.00	1.30	2.20	4.75	5.75	0.93
2022-DNA7	M-1A	20.00	3.50	4.75	4.75	2.50	1.58
2022-DNA7	M-1B	20.00	2.55	3.50	4.75	5.00	0.88
2022-DNA7	M-2	20.00	1.50	2.55	4.75	7.00	1.04
2022-HQA1	M-1A	20.00	3.75	5.00	5.00	2.10	0.51
2022-HQA1	M-1B	20.00	2.60	3.75	5.00	3.50	0.44
2022-HQA1	M-2	20.00	1.25	2.60	5.00	5.25	1.60
2022-HQA1	B-1	20.00	0.75	1.25	5.00	7.00	4.86
2022-HQA1	B-2	20.00	0.25	0.75	5.00	11.00	9.41
2022-HQA2	M-1A	20.00	3.00	4.60	4.60	2.65	0.73
2022-HQA2	M-1B	20.00	2.00	3.00	4.60	4.00	0.77
2022-HQA2	M-2	20.00	1.25	2.00	4.60	6.00	1.08
2022-HQA3	M-1A	20.00	3.00	4.95	4.95	2.30	0.70
2022-HQA3	M-1B	20.00	2.00	3.00	4.95	3.55	0.76
2022-HQA3	M-2	20.00	1.25	2.00	4.95	5.35	1.04
2023-DNA1	M-1A	20.00	3.25	5.25	5.25	2.10	1.00
2023-DNA1	M-1B	20.00	2.55	3.25	5.25	3.10	0.99
2023-DNA1	M-2	20.00	1.50	2.55	5.25	5.50	1.00
2023-DNA1	B-1	20.00	0.75	1.50	5.25	8.15	1.06
2023-DNA2	M-1A	20.00	3.25	5.50	5.50	2.10	1.02
2023-DNA2	M-1B	20.00	2.50	3.25	5.50	3.25	1.00
2023-DNA2	M-2	20.00	1.50	2.50	5.50	5.70	0.97
2023-DNA2	B-1	20.00	1.00	1.50	5.50	7.60	0.95
2023-HQA1	M-1A	20.00	3.00	4.00	4.00	2.00	1.70
2023-HQA1	M-1B	20.00	2.00	3.00	4.00	3.50	0.95
2023-HQA1	M-2	20.00	1.50	2.00	4.00	5.50	0.96
2023-HQA2	M-1A	20.00	4.30	5.75	5.75	2.00	1.30
2023-HQA2	M-1B	20.00	3.00	4.30	5.75	3.35	1.13
2023-HQA2	M-2	20.00	2.60	3.00	5.75	3.85	1.09
2023-HQA3	A-1	20.00	4.25	5.25	4.25	1.85	0.82
2023-HQA3	M-1	20.00	3.25	4.25	4.25	1.85	1.39
2023-HQA3	M-2	20.00	2.25	3.25	4.25	3.35	1.23
2024-DNA1	A-1	20.00	4.25	5.50	4.25	1.35	0.77
2024-DNA1	M-1	20.00	2.90	4.25	4.25	1.35	1.22
2024-DNA1	M-2	20.00	2.20	2.90	4.25	1.95	1.17
2024-DNA2	A-1	20.00	4.00	5.50	4.00	1.25	0.79

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Deal	Tranche	Mat	Attach	Detach	Min CE	Obs Sprd	Mod Sprd
2024-DNA2	M-1	20.00	2.50	4.00	4.00	1.20	1.24
2024-DNA2	M-2	20.00	2.20	2.50	4.00	1.70	1.19
2024-DNA3	A-1	20.00	4.00	5.00	4.00	1.05	0.68
2024-DNA3	M-1	20.00	2.80	4.00	4.00	1.00	1.09
2024-DNA3	M-2	20.00	2.25	2.80	4.00	1.45	1.13
2024-HQA1	A-1	20.00	4.40	5.50	4.40	1.25	0.82
2024-HQA1	M-1	20.00	3.30	4.40	4.40	1.25	1.40
2024-HQA1	M-2	20.00	2.25	3.30	4.40	2.00	1.25
2024-HQA2	A-1	20.00	4.00	5.00	4.00	1.25	0.64
2024-HQA2	M-1	20.00	3.00	4.00	4.00	1.20	1.18
2024-HQA2	M-2	20.00	2.30	3.00	4.00	1.80	1.10
2025-DNA1	A-1	20.00	4.20	5.75	4.20	0.95	0.70
2025-DNA1	M-1	20.00	2.60	4.20	4.20	1.05	1.16
2025-DNA1	M-2	20.00	1.95	2.60	4.20	1.35	1.18
2025-HQA1	A-1	20.00	4.20	5.75	4.20	0.95	0.70
2025-HQA1	M-1	20.00	2.85	4.20	4.20	1.15	1.25
2025-HQA1	M-2	20.00	2.10	2.85	4.20	1.65	1.21

Table A.2: Model-Minus-Observed CRT Spreads (Baseline and Alternative Models By Year)

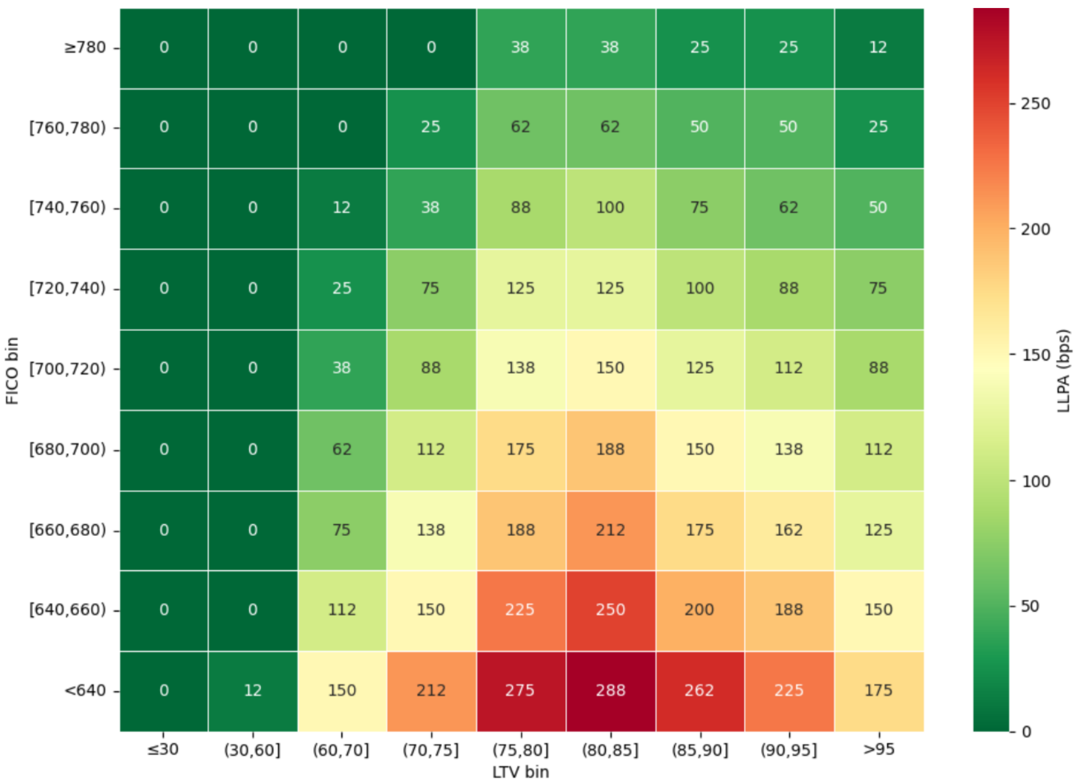
	(1)	(2)	(3)	(4)	(5)
2013	-332.4***	-339.8***	-332.5***	-332.5***	-405.3***
2014	-262.6***	-257.0***	-262.7***	-262.6***	-313.5***
2015	-254.4***	-260.3***	-265.4***	-265.4***	-293.6***
2016	-207.6***	-204.8***	-211.7***	-211.9***	-213.2***
2017	-139.8***	-148.4***	-141.5***	-141.7***	-138.2***
2018	-96.6***	-106.5***	-107.0***	-101.6***	-94.1***
2019	-19.8	4.4	-106.1***	-97.0***	-112.1***
2020	-34.7	-51.7	-150.6***	-144.4***	-210.2***
2021	-40.9*	-3.4	-104.9***	-104.0***	-135.6***
2022	-281.6***	-284.1***	-187.7***	-184.1***	-281.5***
2023	-216.2***	-215.5***	26.4	30.1	-175.0***
2024	-35.8***	-36.4***	172.6***	177.4***	-22.1**
2025	-15.1*	-15.1*	277.7***	283.8***	-21.7**
2013 × *Junior*	-276.7**	-294.8**	-302.0**	-303.1**	-288.0**
2014 × *Junior*	-209.9***	-219.7***	-220.0***	-220.5***	-209.8***
2015 × *Junior*	219.9**	53.3	-355.5***	-340.7***	-389.0***
2016 × *Junior*	452.5***	195.5**	-445.9***	-432.6***	-459.7***
2017 × *Junior*	568.9***	322.3***	-264.2***	-253.8***	-313.8***
2018 × *Junior*	378.4***	301.0***	-158.8***	-142.7***	-272.4***
2019 × *Junior*	611.9***	470.5***	-261.0***	-247.5***	-357.3***
2020 × *Junior*	493.5***	394.2***	-283.2***	-271.6***	-378.6***
2021 × *Junior*	208.3***	243.3***	-186.6***	-178.8***	-305.4***
2022 × *Junior*	-93.3	-87.4	-240.7***	-236.2***	-414.0***
2023 × *Junior*	-470.5***	-472.8***	-306.8***	-302.3***	-409.9***
R^2 (%)	58.7	51.6	63.7	63.1	61.6
Average	-3.1	-47.4	-254.1	-246.5	-343.0

The indicator variable *Junior* equals one for tranches with an attachment point $\leq 1\%$. Years 2024 and 2025 contain no junior tranches. Column (1) is the baseline recursively-estimated model. Column (2) uses the full sample for estimation. Column (3) zeroes out market price of risk (MPR) coefficients associated with house price growth shocks. Column (4) also zeroes MPR coefficients for corporate credit return shocks. Column (5) zeroes all MPR coefficients. Stars indicate robust (HC1) significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

A.6 Loan Level Price Adjustments

Table A.3 shows the current loan level price adjustment grid, as of June 2025. This is the grid that has been in place since the May 2023 LLPA reform.

Figure A.3: Loan Level Price Adjustment Grid Post May 2023 Reform



Notes: Loan Level Price Adjustment grid as of June 2025. The source is the Freddie Mac Single-Family Seller/Servicer Guide, Exhibit 19 (Credit Fees), Bulletin 2025-07, June 4, 2025.

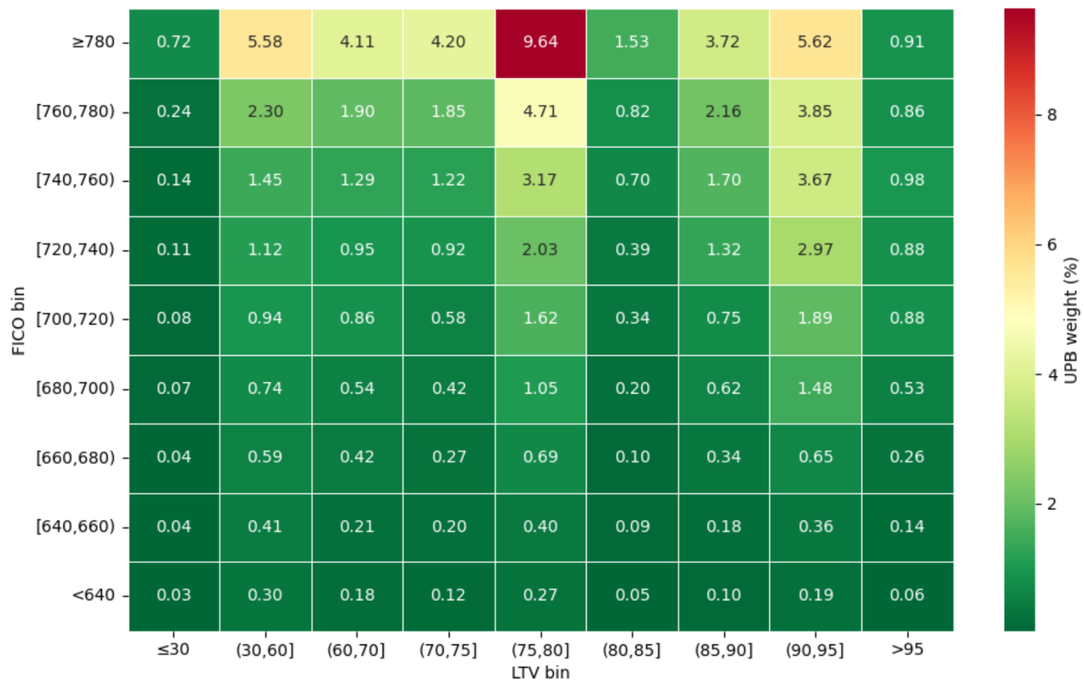
Table A.4 shows the empirical distribution of mortgage UPB by (FICO,LTV) cell in the 2024 mortgage vintage.

A.7 Alternative Models of Income Risk

This appendix considers two alternative models of income risk that generate a much larger FICO slope in the LLPAs. They match the observed slope of basis points between the lowest-FICO and highest-FICO bins for the LTV (70,75] group. We also consider an alternative model of housing risk that generates a smaller LTV slope for the $[720, 740)$ FICO group.

The first alternative model changes $\Lambda_1^{cr,cs}$ from the benchmark value of 13.2 to a new value of 39.6. This change results in a risk-neutral drift of aggregate income growth of 28 bps per month instead of 41 bps in the benchmark model. This in turn generates much more income-driven default risk for low-FICO households than for high-FICO households, enough to reproduce the slope in

Figure A.4: Empirical Borrower Distribution by FICO and LTV (% of loans) for the 2024 vintage.



the data. For the $(70, 75]$ bucket, the model-implied FICO differential becomes 214 basis points, close to the 212 basis points in the data. However, this comes at the cost of the alternative model entirely missing the dynamics of the credit risk premium. In the benchmark model, a regression of the credit excess return on the lagged credit spread results in a slope of 13.2, which matches the data. In the alternative model, that slope is 39.6, resulting in a credit risk premium that is far too volatile and cyclical.

The second alternative model modifies the mapping from FICO scores to initial asset levels, which is given by (40) in the benchmark calibration, and replaces it by:

$$A_0^i = \left(0.5 + \max\left\{\frac{\text{FICO}^i - 500}{17.5}, 0\right\}\right) \cdot K_0^i$$

By giving low-FICO borrowers a lot fewer assets, this model increases their exposure to aggregate income risk, relative to the exposure of high-FICO borrowers, enough to reproduce the observed FICO slope in LLPAs. For instance, the resulting FICO slope of the $(70, 75]$ bucket is 215 basis points, very similar to the data. However, this model generates default rates that are too high relative to the historical sample. Specifically, the UPB-weighted RMSE between observed default rates in the model and the data is 2.3%, while it was far lower at 1.4% in the main model.

For the LTV dimension, setting the net rental yield to be 2.85% per year generates an LTV slope of 120 basis points, close to the observed 125 basis points for the $[720, 740)$ FICO group.

However, this is far below the net rental yield of 4.5% per year, estimated by Eisfeldt and Demers (2015) in U.S. data.

A.8 Structure of G-Fees

Table A.3 describes the components of the guarantee fee charged to borrowers in conforming mortgages, and passed on from loan servicers to the GSEs when the loans are securitized into agency mortgage-backed securities.

Table A.3: Guarantee Fee (G-Fee) Structure Summary

Component	Description
Base G-Fee	- Uniform fee applied to all loans regardless of borrower risk. - Collected monthly as a fixed percentage of unpaid principal balance (UPB). - Typical level: 30–50 basis points (bps) annualized.
Loan-Level Price Adjustments (LLPAs)	- Risk-based add-ons based on loan characteristics: credit score, LTV, loan purpose, product type, etc. - Charged as upfront fees at origination (quoted in % of UPB), but often converted into annualized bps for pricing. - Can range from 0 to 300+ bps upfront, depending on risk.
G-Fee	- Sum of Base G-Fee and annualized LLPAs.
Collection Method	- Ongoing is paid monthly as part of borrower's mortgage payment. Upfront is paid as a lump sum payment at loan origination. - Applied to outstanding UPB; declines with amortization or prepayment.
Use of Proceeds	- Funds GSE credit guarantee operations, capital build-up, and CRT payments. - Supports coverage of expected credit losses, admin costs, capital charge, and uncertainty buffer.