

26:711:651: Linear programming

Fall 2018

1WP Room 402, Newark campus

Thursdays 3:00-5:50pm

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Office Hours: By appointment

COURSE DESCRIPTION

The aim of this course is to introduce graduate students to linear programming and its extensions with an emphasis on the mathematical formulations, algorithms and solutions for practical problems arising in business research and operations research including supply chains, network science, marketing and finance. The class will also include programming exercises with MATLAB (or Octave), AMPL and Excel software for formulating and solving real world problems.

COURSE MATERIALS

- Required text:

- M.S. Bazaraa, J.J. Jarvis, H.D. Sherali, *Linear programming and network flows*, 4th Edition, Wiley, 2010.

(This is our main text).

- Recommended text:

- R. Fourer, D. Gay, B. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, 2nd Edition, Boyd & Fraser Publishing Company, 2002.

(This is important for learning AMPL, a modeling language for linear and non-linear programming.

It is available in pdf format from the site <https://ampl.com/resources/the-ampl-book/>.

- Check Blackboard (blackboard.rutgers.edu) and your official Rutgers email account regularly.

LEARNING GOALS AND OBJECTIVES

- This course is designed to help students develop skills and knowledge in the following areas:

- *Developing skills: Mathematical background for linear programming, ability to interpret, formulate, solve and analyze linear programs arising in practical applications.*
- *Developing knowledge base: Students will get more familiar with linear programming tools so that they can use it in their own research both in social sciences and applied sciences. The students will have a good understanding of both the theory and practice for solving linear programming problems arising in the business research.*

- Students who complete this course will demonstrate the following:

- *Ability to apply linear programming techniques for solving and modeling some fundamental decision making problems arising in the daily business life.*

- Students develop these skills and knowledge through the following course activities and assignments:

- Students will complete homework assignments, a midterm and a final exam. Lectures will be interactive with students, allowing them to interact with their peers in collaborative class discussions.

PREREQUISITES

- A good knowledge of undergraduate level linear algebra and calculus techniques for optimization such as matrix computations, optimality conditions and Lagrange multipliers.
- Familiarity with at least one software tool that can be used for numerical computations (such as Excel, R, MATLAB, Python or equivalent).

ACADEMIC INTEGRITY

I do NOT tolerate cheating. Students are responsible for understanding the RU Academic Integrity Policy (https://slwordpress.rutgers.edu/academicintegrity/wp-content/uploads/sites/41/2014/11/AI_Policy_2013.pdf). I will strongly enforce this Policy and pursue *all* violations. On all examinations, students must sign the RU Honor Pledge, which states, “On my honor, I have neither received nor given any unauthorized assistance on this examination or assignment.” Don’t let cheating destroy your hard-earned opportunity to learn. See business.rutgers.edu/ai for more details.

ATTENDANCE AND PREPARATION POLICY

- Expect me to attend all class sessions. I expect the same of you. If I am to be absent, my department chair or I will send you notice via email and Blackboard as far in advance as possible. If you are to be absent, report your absence in advance at <https://sims.rutgers.edu/ssra/>. If your absence is due to religious observance, a Rutgers-approved activity, illness, or family emergency/death and you seek makeup work, also send me an email with full details and supporting documentation within 3 days of your first absence.

- For weather emergencies, consult the campus home page. If the campus is open, class will be held.

- Expect me to arrive on time for each class session. I expect the same of you.

- Expect me to remain for the entirety of each class session. I expect the same of you.

- Expect me to prepare properly for each class session. I expect the same of you. Complete all background reading and assignments. You cannot learn if you are not prepared. The minimum expectation is that for each class, you have prepared by studying for at least twice as many hours.

- Expect me to participate fully in each class session. I expect the same of you. Stay focused and involved. You cannot learn if you are not paying attention.

CLASSROOM CONDUCT

- Do not call out in class. If you have a question, raise your hand.

- No food or drink in class.
- No side conversations or use of cell phones in class.
- Use the bathroom before class to avoid the necessity of bathroom breaks.

EXAM DATES AND POLICIES

There are **two** exams in this course. The exam dates are:

- Midterm exam: **Thursday October 25th**
- Final exam: **Thursday December 6th**

During the exam, the following rules apply:

- If you have a disability that influences testing procedures, provide me an official letter from the Office of Disability Services **at the start of the semester**.
- No cell phones or other electronics are allowed in the testing room.
- You must show a valid Rutgers photo ID to enter the room and to turn in the exam.
- Alternate seating; do not sit next to another student or in your usual seat. A new seat will be assigned on a random basis.
- Use the bathroom prior to the exam start; bathroom breaks, if essential, will be escorted.
- Your exam will not be accepted unless you sign the Honor Pledge

Make-up exam policy:

- **Allowances for make-up exams** “Make-up” exams are allowed only for those students whose absence on a class exam date was due to a legitimate illness or emergency (i.e., circumstances beyond their control). MSIS Department Coordinator Office ultimately determines what does or does not constitute a “legitimate” illness/emergency.
- **Procedures for obtaining authorization to take a make-up exam:** If your absence is due to illness, the MSIS Department Coordinator Office will require you to provide them with a document from your doctor indicating that you were indeed sick that day. If you do not provide the MSIS Department Coordinator Office with a doctor's note, they cannot in turn provide me with the proper authorization to allow a make-up exam.

GRADING POLICY

There will be 6-7 homework assignments, issued on a Thursday after class. Homeworks will include programming exercises as well as conceptual questions. Each assignment should be submitted online on Blackboard, in any legible format (scans of handwritten solutions are fine). All homework assignments are to be completed on your own. You are welcome to discuss homework problems with me during office hours. Should a student copy the homework of another student, both of them will get only half grade.

All homework assignments and exams will be graded on a scale from A to F. To calculate your final grade, the individual grades will be converted to a number, and a weighted average will be taken according to the following weights.

50% Homework Assignments
 20% Midterm Exam
 30% Final Exam

Your final grade is not subject to negotiation. If you feel I have made an error, submit your written argument to me within one week of receiving your final grade. Clarify the precise error I made and provide all due supporting documentation. If I have made an error, I will gladly correct it. But I will adjust grades only if I have made an error. I cannot and will not adjust grades based on consequences, such as hurt pride, lost scholarships, lost tuition reimbursement, lost job opportunities, or dismissals. Do not ask me to do so. It is dishonest to attempt to influence faculty in an effort to obtain a grade that you did not earn, and it will not work.

COURSE SCHEDULE

<u>Date</u>	<u>Topic</u>
Sep 6 th	<p>Formulations and definitions. <i>Topics: Introduction to linear programming (LP), history of LP, some realworld problems including investment under taxation, diet problem, applications to manufacturing and scheduling, introduction to software (AMPL, Excel and MATLAB-like languages)</i> Reading: Bazaraa et al (Chapter 1), AMPL reference book (Chapter 1), lecture notes</p>
Sep 13 th	<p>Review of linear algebra <i>Topics: Quick review of basic facts from matrices: block matrices, transpose, inverse, determinant, Gaussian elimination and LUP factorization, Cholesky factorization, matrix notation for linear programs.</i> Reading: AMPL book (Chapter 2), Bazaraa et al (Chapter 2), lecture notes</p>
Sep 20 th	<p>Review of convex sets and functions, polyhedra <i>Topics: Convex sets and function, separation theorem, convex cones, hyperplanes, polyhedra, faces, facets, edges, and extreme points and rays, Caratheodory's theorem. Convex polytopes, convex hulls.</i> Reading: AMPL book (Chapter 2), Bazaraa et al (Chapter 2), lecture notes</p>
Sep 27 th	<p>The simplex method <i>Topics: The basic feasible solutions and connection to extreme points, basic and non-basic variables, the simplex iteration, reduced costs and exchange of basic and nonbasic variables, optimality criterion. The tableau notation. The so-called revised simplex method (matrix formulation of the simplex method). Use of LU factorization, and efficient updating of it when exchanging basic and non-basic variables (Sherman-Morrison-Woodbury formula and related topics). Finding an initial basic solution, the Big M method. Unbounded, and infeasible problems, and their detection degeneracy and non-unique solutions, possib of cycling and methods to avoid it computational complexity of simplex, Klee-Minty examples.</i> Reading: Bazaraa et al (Chapters 3-5), lecture notes</p>
Oct 4 th	<p>Duality and complementary slackness <i>Topics: The dual problem, weak duality theorem, strong duality using the tableau. Dual of the diet and the transportation problems, economic interpretation of duality, the arbitrage theorem in finance and duality. Convex sets and functions, separation theorem, Farkas lemma, proof of duality theorem based on Farkas lemma*.</i> Reading: lecture notes, Bazaraa et al. (Chapter 6), lecture notes</p>

- Oct 11th **The dual simplex method**
Topics: The dual simplex method. Complementary slackness theorem and its consequences, economic interpretation primal-dual simplex methods dual of general linear programs.
 Reading: lecture notes, Bazaraa et al. (Chapter 6), lecture notes
- Oct 18th **The Wolf-Dantzig and Benders decompositions**
Topics: The Dantzig-Wolfe decomposition. Benders decomposition.
 Reading: Bazaraa et al. (Chapter 7), lecture notes
- Oct 25th **Mid-term exam**
- Nov 1st **Computational complexity and Interior point algorithms**
Topics: logarithmic barrier and potential functions, path following primal-dual methods, computational complexity analysis
 Reading: lecture notes
- Nov 8th **Introduction to game theory**
Topics: concept of matrix games, optimal strategies and the min-max theorem, connection to duality in linear programming Nash equilibrium, some economic applications.
 Reading: lecture notes
- Nov 15th **Network flows**
Topics: The general minimum cost network flow problem. Special cases such as the transshipment problem, transportation problem, maximum flow problem, assignment problem and shortest path problem. Interpretation of bases and pivots in network problems: spanning trees, and bases.
 Reading: Bazaraa et al. (Chapters 9, 10 and 12), chapter 15 of AMPL book, lecture notes
- Nov 20th **The network simplex method**
Topics: The network simplex method. Primal-dual simplex method and its application to network flow problems. The transportation problem, simplex and primal-dual simplex methods. Assignment problem and the Hungarian method maximum flow problem and Ford-Fulkerson augmenting path algorithm and Goldberg's push-relabel algorithm. Network flow problems with loss or gain on arcs or on nodes, applications to currency arbitrage. Dijkstra's shortest algorithm and dynamic programming and connection primal-dual method.
 Reading: Bazaraa et al. (Chapters 9, 10 and 12), chapter 15 of AMPL book, lecture notes
- Nov 29th **Introduction to Integer programming**
Topics: Integer Programming and mixed integer programming and contrast to linear programming Fundamental difficulty of solving integer programs. The branch and bound method: A simple case: The knapsack problem. The branch and bound method: the more general case and use of the dual simplex method in solving integer programs. Relaxed Lagrangian approach, applications to the traveling salesman problem

Reading: Chapter 20 of AMPL book, lecture notes

Dec 6th

Final exam

SUPPORT SERVICES

If you need accommodation for a *disability*, obtain a Letter of Accommodation from the Office of Disability Services. The Office of Disability Services at Rutgers, The State University of New Jersey, provides student-centered and student-inclusive programming in compliance with the Americans with Disabilities Act of 1990, the Americans with Disabilities Act Amendments of 2008, Section 504 of the Rehabilitation Act of 1973, Section 508 of the Rehabilitation Act of 1998, and the New Jersey Law Against Discrimination. <https://ods.rutgers.edu>

If you are a military *veteran* or are on active military duty, you can obtain support through the Office of Veteran and Military Programs and Services. <http://veterans.rutgers.edu/>

If you are in need of *mental health* services, please use our readily available services.
Rutgers University-Newark Counseling Center: <http://counseling.newark.rutgers.edu/>

If you are in need of *physical health* services, please use our readily available services.
Rutgers Health Services – Newark: <http://health.newark.rutgers.edu/>

If you are in need of *legal* services, please use our readily available services: <http://rusls.rutgers.edu/>

If you are in need of additional *academic assistance*, please use our readily available services.
Rutgers University-Newark Learning Center: <http://www.ncas.rutgers.edu/rlc>
Rutgers University-Newark Writing Center: <http://www.ncas.rutgers.edu/writingcenter>