Risk-Adjusted Stock Information from Option Prices

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Abstract

This paper investigates a risk-adjusted approach for joint estimation of implied expected stock return and implied volatility from market observed option prices. There are two findings from our investigation. First, our result shows that investors in option markets have a higher expectation of stock return in the short-term, but a lower expectation of stock return in the long-term. This finding is robust to different measures of option prices such as the European price and the American price. We control for market friction proxies such as volume, open interest and bid-ask spread and the results are robust. This term structure also persists for combined call and put options. Finally, the empirical investigation shows that a combination of our implied expected return and implied volatility with Black-Scholes implied standard deviation (ISD) provides a better model, than ISD alone to forecast future volatility of stocks.

JEL classification G11, G12, G13

Keywords implied return, implied cost of equity, risk-adjusted pricing

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**Introduction**

This research investigates a risk-adjusted method for joint estimation of implied expected stock return (or simply the implied return, $\mu$) and implied volatility ($\sigma$) from market prices of options. We use a discrete time approach that retains the underlying expected return in the option pricing equation. Option pricing models of Sprenkle (1961), Ayres (1963), and Boness (1964) had implicitly or explicitly assumed some form of risk-adjusted model such that the investors buy and hold the options until maturity to extract the option implied return, which then could be linked to the stock return. However, none of these models provides an adequate theoretical structure to determine the implied return values. The Black-Scholes (1973), models the option price by taking advantage of the interesting feature that a certain portfolio of the stock and the option can cancel out the unknowns namely the implied stock return and the implied option return in continuous time. However, if our objective is to extract implied expected return given the market price of options, we form the corresponding risk-adjusted valuation model that will retain the expected returns in the pricing model. A paper by Galai (1978) compares the risk-neutral and the risk-adjusted approach of option pricing, in which the author shows that if we use the risk-adjusted model then it will retain the stock implied return in the pricing equation. Our approach parallels this approach. However, there are at least three differences between our approach and his approach. First, Galai compares the properties of implied option expected return derived from risk-adjusted model with the risk-neutral model, whereas we derive a relationship between the implied expected stock return and expected option return in continuous time and then link that with the risk-adjusted model. Second, we derive a discrete time version of equations for covariance of option return and stock return, and variance of stock return. Finally, we compute the implied stock return and volatility from observed market price of options whereas his paper assumes a range of implied stock expected returns as given and then uses the equations to compute a range of implied option expected returns.
Heston (1993b) provides another related paper that studies the properties of the risk-neutral option pricing. In this paper, Heston shows a log-gamma formula, which depends on the mean return parameter but is independent of volatility, the scale parameter. If this distribution holds then option prices will be insensitive to volatility which contrasts with many findings that implied volatility has useful information to explain realized volatility. Nonetheless, Heston’s paper shows the possibility of retaining the expected return parameter in the model with suitable adjustments to the pricing equation. A more recent research by McNulty (2002) uses a heuristic approach to compute the ‘real cost of equity capital’ using option prices. Their finding of higher implied expected return in the short term and lower implied expected return in the long term matches with our finding; however, their approach lacks the theoretical setting to arrive at the result. On the other hand, our paper provides the necessary theoretical support for this term structure finding. Another recent paper, which computes the cost of equity from option prices, is Camara et al. (2007). Their approach uses a specific (DPRA) utility structure. Furthermore, their approach requires an intermediate parameter that needs to be computed using options of all firms before they compute the implied expected return of any individual firm. In contrast to their paper, we follow a preference-free pricing method similar to Black-Scholes and Galai to extract the implied expected returns and we do not need information about other stock options to compute the implied expected return of a specific stock. Interestingly, similar to our finding, Camara et al. (2007) results also show that the short-term expected returns are higher than the long-term expected returns. However, unlike their paper, we do many robustness tests for the existence of this term structure. We further show the forecastability of realized volatility of stocks using our estimated parameters.

To validate the robustness of our finding, we examine the influence of market friction proxies such as the option open interest, volume, and bid-ask spread on the term structure of implied expected return. We show, these proxies do not explain this term structure. We
further examine both the European option price, which is reverse computed from implied volatility in our input data, and the American option price, as the mid point of bid-ask spread in the option pricing equation. In both cases, we find a similar term structure of implied expected return. We also find the term structure of expected return remains for deep-in and deep-out of the money call options.

We further examine the information content of our implied expected return and volatility to forecast future volatility of stock returns. A vast body of literature exists that investigates the forecasting capability of implied volatility from option prices. A recent survey by Granger and Poon (2005) categorizes the future volatility forecast, based on past research, into four methods namely: historical volatility method, ARCH and GARCH models, stochastic volatility models, and Black-Scholes implied volatility method. Their overall ranking suggests that Black-Scholes implied volatility provides the best forecast of future volatility among all these methods. Therefore, we choose Black-Scholes implied volatility (ISD) as the benchmark, and compare the information content of our implied expected return (μ) and implied volatility (σ) with the ISD.

We find there are at least three advantages of using μ, σ and ISD, than using ISD alone in forecasting the volatility, more so for lower days-to-maturity. First, when we use all the three variables (μ, σ and ISD) and their second-order terms in the volatility forecast regression, the coefficient of intercept becomes insignificant. However, the intercept term is significant when we use only ISD and its second-order term. This shows even though ISD explains a significant portion of realized volatility, there is a large unexplained intercept in this regression. Second, for lower days-to-maturity, the coefficients of μ, σ,
and their second-order terms are significant even in the presence of Black-Scholes implied volatility. Finally, a likelihood ratio test rejects the null hypothesis that the restricted model with ISD and its second-order term is better than the unrestricted model with all the three variables and their second-order terms.

The reminder of this paper is organized as follows. Section 2 presents the risk-adjusted discrete time model that retains the stock expected return in the option pricing equation. Section 3 presents the joint estimation of implied expected stock return and implied volatility. Section 4 discusses the empirical results of our estimation. Section 5 discusses future volatility forecasting comparison. Section 6 provides the concluding remarks.

2. The Option Valuation Model

It is well known that the Black-Scholes model can be used to compute implied volatility and not implied expected return of the underlying stock due to the fact that no-arbitrage argument renders a preference-free model and hence contains no such parameter. In this sub-section, we demonstrate that such parameter can be re-discovered via an “equilibrium” pricing approach similar to Samuelson (1965) and Sprenkle (1961). Let the stock price follow the usual log normal process under the physical measure:

\[
\frac{dS}{S} = \mu dt + \sigma dW
\]

where the annualized instantaneous expected return is \( \mu \) and the volatility is \( \sigma \). The classical economic valuation theory states that any price today must be a properly discounted future payoff:

\[
C_t = E_t[M_{t,T}C_T]
\]
where $M_{t,T}$ is the pricing kernel, also known as the marginal rate of substitution, between time $t$ and time $T$. The usual risk neutral pricing theory developed by Cox and Ross (1976) performs the following change of measure:

$$
C_t = E_t[M_{t,T}C_T]
$$

(3)

$$
= E_t[M_{t,T}]E_t^Q[C_T]
$$

$$
= \begin{cases} e^{-r(T-t)}E_t^Q[C_T] & \text{if interest rate is constant} \\
F_{t,T}E_t^F(T)[C_T] & \text{if interest rate follows a random process}
\end{cases}
$$

where $Q$ represents the risk neutral measure and $F(T)$ represents the $T$-maturity forward measure and, $P_{t,T}$ is the risk free zero coupon bond price of $\$1$ paid at time $T$.

In this paper, we perform the change of measure in the other direction. That is,

$$
C_t = E_t[M_{t,T}C_T]
$$

(4)

$$
= E_t[C_T]E_t^X[M_{t,T}]
$$

$$
= E_t[C_T]e^{-k(T-t)}
$$

where $X$ represents the measure where the option price serves as a numeraire. We then assume that the $X$-measure expectation of the pricing kernel takes a form of continuous discounting. Now, we can derive our option pricing formula as:

$$
C_t = e^{-k(T-t)}E_t[\max\{S_T - K, 0\}]
$$

(5)

$$
= e^{-k(T-t)}\int_0^\infty S_T \phi(S_T) dS_T - K \int_0^\infty \phi(S_T) dS_T
$$

$$
= e^{(\mu-k)(T-t)}S_tN(h_1) - e^{-k(T-t)}KN(h_2)
$$

Note that the last line of the equation is a classical result of economic valuation.
where $t$ and $T$ are the current time and maturity time of the option, and $K$ is the strike price of the option and

$$h_1 = \frac{\ln S - \ln K + \left( \mu + \frac{1}{2} \sigma^2 \right)(T - t)}{\sigma \sqrt{T - t}}$$

$$h_2 = h_1 - \sigma \sqrt{T - t}$$

To derive a pricing formula that contains $\mu$, we need the following propositions. These propositions describe how implied return and volatility can be simultaneously estimated from option prices.

**Proposition 1.** Assume stock price $S$ follows a geometric Brownian motion with an annualized expected instantaneous return of $\mu$ and volatility of $\sigma$. Let a call option on the stock at any point in time $t$ is given by $C(S,t)$ that matures at time $T$. Let $k$ is the annualized expected instantaneous return on this option. Then assuming no-arbitrage argument for a small interval of time $\Delta t$, the relationship between $\mu$ and $k$ for this interval can be given by:

$$k = r + \beta(\mu - r)$$  \hspace{1cm} (6)$$

where

$$\beta = \frac{\text{cov}(r_C, r_S)}{\text{var}(r_S)}$$  \hspace{1cm} (7)$$

and $r_S = \Delta S / S$ and $r_C = \Delta C / C$ are two random variables representing the stock return and call option return respectively during the period $\Delta t$. And, $r$ is the annualized constant risk-free rate for the period of the option. Proposition 1 can be proved without assuming the CAPM.
**Proof.** See Appendix A.

Equation (6) holds for a small interval of time $\Delta t$. We assume the distributions of stock return, $r_s$, and option return, $r_c$, are stationary over the period of the option. This implies the annualized instantaneous expected return and variance over a small interval of time and the annualized instantaneous expected return and variance over the discrete time (from $t$ to $T$) will be same. This also implies $\beta$ is constant over this period, which means the linear relationship between $k$ and $\mu$ as in equation (6) is valid over the life of the option from current time $t$ to maturity time $T$. Since our approach will be pricing the option in a discrete setting, we approximate the $\beta$ over the discrete time from $t$ to $T$ as:

\[
\beta = \frac{\text{cov} \left( \frac{C_T}{C_t}, \frac{S_T}{S_t} \right)}{\text{var} \left( \frac{S_T}{S_t} \right)} = \frac{S_t \text{cov} \left( C_T, S_T \right)}{C_t \text{var} (S_T)}
\]

Equation (6) with equation (7), in continuous time, and equation (6) with equation (7a) in discrete time can also be proved using the CAPM without the Black-Scholes no-arbitrage argument. However, for these two equations to hold it is not necessary that the CAPM should hold. The assumptions of the CAPM are much stronger so that all return distributions are stationary, however here we need only the stationarity of the stock and the option return along with Black-Scholes no-arbitrage argument to obtain these two equations. Hence stationarity assumption of $r_s$ and $r_c$ is a weaker assumption than what is needed for CAPM. Further Galai (1978) shows many similarities between the continuous time and discrete time properties of $r_c$ that support our assumption of stationarity of distribution.

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6 This can be easily seen by integrating both side of equation (6) from $t$ to $T$. 
Also we note, the right hand side of (7a) is a close approximation of $\beta$ when we assume stationarity of $r_s$ and $r_c$.

Equation (5) is obtained based on the assumption that the expected return of the option $k$, expected return of the stock $\mu$ and volatility $\sigma$ are constants. We have also assumed that the stock price follows a geometric Brownian motion. Combining equations (7a) and (5), we arrive at the following proposition.

**Proposition 2:** The $\beta$, based on the life of the option can be written as:

$$
\beta = \frac{S_t \left[ e^{\sigma^2(T-t)} N(h_3) - \frac{K}{S_t} e^{-\mu(T-t)} \left( N(h_1) - N(h_2) \right) - N(h_1) \right]}{C_t \left[ e^{\sigma^2(T-t)} - 1 \right]}
$$

where

$$
h_3 = \ln S - \ln K + \left( \mu + \frac{1}{2} \sigma^2 \right) (T - t)
$$

**Proof:** See Appendix B.

It should be noted that $\mu$ and $k$ in continuous time as given in equation (6) and in the discrete time as given in (5) are the same with the assumption of stationarity of $r_s$ and $r_c$. Moreover, we do not use the distributional properties of the market return $r_M$ to obtain (8).
Using (8), (5), and (6) we can solve for the call price $C_t$ explicitly in terms of the known values: stock price ($S_t$), strike price ($K$), risk free rate ($r$), time-to-maturity ($T - t$), and two important unknown parameters: expected stock return $\mu$ and volatility $\sigma$. If we observe the values of two or more call options, with same time-to-maturity with different strike prices, we can then simultaneously solve for $\mu$ and $\sigma$.\(^7\)

### 3. Estimation

To extract implied return from option prices we use the end-of-day OptionMetrics transaction data of options of all stocks listed on NYSE, NASDAQ, and AMEX for the last business day of every month during January 1996 – April 2006. This data file contains the end-of-day stock CUSIP, strike price, offer, bid, volume, open interest, days-to-maturity, and Black-Scholes implied volatility for each option. From this dataset, we exclude all put options, not near-the-money call options, and options with zero trading volume. We define near-the-money option as any option whose stock price/strike price falls between 0.95 and 1.05. We take only near-the-money options to avoid the volatility smile. In this paper, the results are based on the last business day observations for each calendar month. Taking any other day of the month produces similar results. For example, we verified our results by taking first working day, second Thursday, and third Friday of every month. The results are similar. For the corresponding stocks, we obtain daily prices and returns from CRSP. Returns are needed to compare the volatility forecasting models as given in section 5. To match the stock price with option records we use the trade date and CUSIP of the stock. For the interest rates, we use the St. Louis Fed’s 3-months, 6-months, 1-year, 2-year, 3-year, and 5-year Treasury Constant Maturity Rates. Assuming a step-function of interest rates, we match the days-to-maturity in the option record with its corresponding constant maturity rate. For example if the days-to-maturity of the option is less than or equal to 3-months we

\(^7\) With prices for options with more than two strike prices, we can find values for $\mu$ and $\sigma$ that produce option prices closest to the observed prices in the least squares sense. A similar least-squares methodology was used by Melick and Thomas (1997).
use 3-months rates, and if the days-to-maturity is between 3-months and 6-months, we use the 6-months rate and so on. We compute closing bid and ask mid point as the closing American price of the option. The option data also have Black-Scholes implied volatility adjusted for any stock dividend during the life of the option. Using this information along with the interest rates we reverse compute the corresponding European option price. If the European option price thus computed is higher than the American price, then we take the American price, else we take the European price as the option price.

3.1 Estimation of Implied Stock Return and Implied Volatility
We jointly estimate the implied expected stock return ($\mu$) and implied volatility ($\sigma$) using the risk-adjusted option pricing model described in previous section. For a given trade date and CUSIP, we have many near-the-money options with same days-to-maturity. We use all these options records to compute implied stock return and implied volatility by a method of grid search to look for the global optima that minimizes the error square. Error is defined as the difference between the theoretical option price using the above implied return and implied volatility, and the market observed option price. We use the implied return search range from 0.0% to 200.00%, and implied volatility search range from 0.0% to 100.00% for the grid search. To compute implied expected stock returns we need two or more records with same key value of trade date, CUSIP, and days-to-maturity. Thus, all the single records for a key value cannot be used to compute implied return and are discarded. We get different implied returns for different days-to-maturity for a given trade date and CUSIP by this process. We also extract the market implied return and market volatility for different days-to-maturity based on S&P500 index option prices, and corresponding S&P500 index levels.

4. Results
4.1 Descriptive Statistics

Table I shows the descriptive statistics of the input data used for our estimation of option implied expected stock return ($\mu$) and implied volatility ($\sigma$). To analyze the results we group the data into different days-to-maturities. Thus the options whose days-to-maturity is less than or equal to 30 days are grouped in 30-days-to-maturity group. The options whose days-to-maturity is greater than 30 days but less then or equal to 60-days are grouped in 60 days-to-maturity groups and so on. As we see from Table I, the number of observations is higher for lower days-to-maturity than for higher days-to-maturity. For example, for 30 days-to-maturity, the number of observations is 11565, and for 720 days-to-maturity, it is 730 observations. Since we take only near-the-money options, the average moneyness mean is around 0.99. In most cases we have around two option records to compute a $\mu$ and $\sigma$ pair. The spread in our data is defined as: (offer – bid)/call price. Interestingly even though all our options are near the money we see the average spread is mostly higher for lower days-to-maturity. Since closer to maturity options are most actively traded, it is not surprising to see the average volume to be higher for lower days-to-maturity. Table II shows the descriptive statistics of $\mu$ and $\sigma$ estimates using all stock options. Table III shows the corresponding descriptive statistics using S&P 500 index options, and Figure I and II show $\mu$ and $\sigma$ graphs for all stocks and S&P500 index option respectively. In these tables and graphs, we see a term structure of $\mu$. For example in Table III for 30 days-to-maturity $\mu$ is 27.04%, whereas for 360 days to-maturity it is 10.85%. The standard deviation of $\mu$ are also lower for higher days-to-maturity. The term structure of $\mu$ implies the expected return is impacted by the time horizon of investment. McNulty et al. (2002) argue that the marginal risk of an investment (the additional risk the company takes on per unit time) declines as a function of square root of time. For example if an investment is expected to be worth $100 in one year but has a projected yearly volatility of 20%, then the expected price has a high likelihood of ranging between $80 and $120 (one standard deviation range in one year). However, if the investment is expected to be worth $100 in
four years and has the same annualized projected volatility then its expected price will likely range between $60 and $140. In other words increasing the holding period from one to four years reduces the per annum risk of the investment by 50%. The falling marginal risk should be reflected in the annual discount rate.\(^8\) Our term structure of \(\mu\) is consistent with this explanation. The data points for the term structure graphs (Figure I and II) are generated by non-parametric spline interpolation using the neighborhood data points. Our approach possibly can be used to estimate the cost of equity for investment projects. Especially estimates of expected return for one-year or more will have lower standard error, which is a necessary condition for this to be useful as an estimate of cost of equity. One of the advantages of our approach is that the expected return of a stock can be computed without using any information of the market portfolio such as the market risk premium. This implies one does not have to define what the ‘market’ consists of, and one does not have to estimate the risk premium of the market, which is required in traditional asset pricing models, to estimate the expected return.

4.2 Influence of Market Friction on the Term Structure of \(\mu\)

In Table I we see the mean of average spread and the mean of average option volume monotonically decrease with days-to-maturity, although the mean of total option open interest does not show any clear pattern.\(^9\) Therefore option spread and option volume could be one possible reason for the term structure of \(\mu\). This experiment is also motivated by the findings of Longstaff (1995). Using S&P100 index options and Black-Scholes (1973) risk-neutral valuation Longstaff shows that the implied cost of the index is significantly higher in the option market than in the stock market. The author also shows the percentage pricing difference between the implied and actual index is directly related to the measures of transaction costs and liquidity such as the option spread, volume, and open interest. To

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\(^8\) This is explained in McNulty et al. (2002).
\(^9\) Average spread is average of \((\text{offer-bid})/\text{option price}\) of all the options used to compute a \(\mu\) and \(\sigma\) pair. Similarly average volume is the average of volumes of options used to compute a \(\mu\) and \(\sigma\) pair.
examine the possible influence of these market friction proxies on the term structure of $\mu$, we regress $\mu$ on transaction cost proxy that is given by the average spread, and liquidity measures that are given by average volume and total open interest. We also control for other finding of pricing biases of Black-Scholes model. These findings include Chiras and Manaster (1978), Macbeth and Merville (1980), and Rubenstein (1985). These studies find three types of pricing bias in Black-Scholes model namely a time to expiration bias, a moneyness bias, and a volatility bias. To control for these biases we include the time to expiration, moneyness (stock price/strike price), and current and first two lagged values of absolute daily returns. To control for volatility bias, we use current and first two lagged values of absolute daily returns instead of implied volatility $\sigma$ since this parameter is jointly estimated with $\mu$, which can induce spurious correlation. Further, as the Longstaff paper, we use number of calls to compute $\mu$ and $\sigma$ as a measure of trading activity, current and lagged daily returns as a measure of path-dependent effects (Leland (1985)).

The results are shown in Table IV. The regression results provide mixed evidence that term structure of $\mu$ is related to the market friction proxies namely spread, volume, and open interest. For example, for 360 days-to-maturity the coefficient of average spread is 0.7868 and is significant. Whereas total open interest and average volume are not significant. Similarly, for other days-to-maturity groups these friction proxies are not significant. Therefore, our evidence shows that friction proxies are not the cause of the term structure of $\mu$.

4.3. Measurement Error and Robustness Check

Our modified risk-adjusted approach can be questionable in a framework with stochastic volatility and jumps, which means we may not be using the exact model of option pricing. Many of the past literature for example Merton (1976a), Cox and Ross (1976a), Hull and White (1987), Scott (1987), and Heston (1993a) extend basic Black-Scholes (B-S) model to incorporate jumps and stochastic volatility. However, the risk-adjusted formulas we use do
not have these adjustments and assumes a lognormal diffusion process. This can create errors-in-variable problem in implied return and implied volatility computation. To minimize the effect of errors-in-variable bias, we have taken options, which are only near-the-money (stock price divided by strike price is between 0.95 and 1.05). Moreover, we do not take options that do not have any trading in a given day. We also separately estimate $\mu$ and $\sigma$ for deep-in-the-money call options where stock price divided by strike price is greater than 1.20, and deep-out-of-the-money call options where stock price divided by strike price is less than 0.90. In both cases, we still get the term structure of $\mu$ (not shown here). Measurement error may be systematically affected by time-to-maturity (Canina and Figlewski(1993)). To mitigate these errors, options with same days-to-maturity are used to compute implied return and implied volatility. It may also be possible to have systematic bias in our computation due to other factors such as the market friction (Longstaff (1995)) proxies. To examine this possibility, we regress $\mu$ on these proxies to show in sub-section 4.2 that they do not have significant effect on the term structure of $\mu$.

Furthermore, our procedure might have problems of computing European option prices from OptionMetrics implied volatility and using that to compute our implied return and implied volatility. As a part of our robustness check, we show even if we use different methods to compute option prices, the term structure of implied expected return remains in our result. For example, in our main result we compute the European price using the OptionMetrics implied volatility adjusted for dividends. If this price is higher than the bid-ask mid point then we take the bid-ask mid point, else we take the European price as the option price for $\mu$ and $\sigma$ estimation. In our robustness check, we compute $\mu$ and $\sigma$ first by taking the European price, and then by taking the bid-ask midpoint price as the option price and we get clear term structures of implied expected return in both cases. Figure III, shows the term structure when we take the European option price as the price of the option using S&P 500 index options.
5. Future Volatility Forecast

5.1 Volatility Forecast Using $\mu$ and $\sigma$

In this section, we want to analyze whether the combined information content of $\mu$, $\sigma$ and Black-Scholes (1973) implied volatility (ISD) provides an improved model to forecast future realized volatility.

In a recent comparison study, Granger and Poon (2005) finds that the Black-Scholes (1973) implied volatility (ISD) provides a more accurate forecast of realized volatilities. In their paper, they show the outcomes of 66 previous studies in this area that uses different methods to forecast the realized volatility. These methods are historical volatility, ARCH, GARCH, ISD, and stochastic volatility (SV). Based on their ranking they suggest that ISD provides the best forecast of future volatility. Despite the added flexibility of SV models, authors find no clear evidence that they provide superior volatility forecasts. Furthermore, they find ISD dominates over time-series models because the market option prices fully incorporate current information and future volatility expectations. Therefore, we choose Black-Scholes implied volatility (ISD) as the benchmark, and compare the information content of our implied expected return ($\mu$) and implied volatility ($\sigma$) with the ISD. To understand the forecastability of realized volatility using $\mu$, $\sigma$ and ISD we plot these time series values in Figure IV and Figure V for 30 days-to-maturity and 180 days-to-maturity respectively for S&P500 index options.

The comparison of information content of ISD over a combination of $\mu$, $\sigma$ and ISD can be evaluated using the following two regressions:

(R1) \[ relVol_{it} = \alpha_0 + \alpha_1 ISD_{it} + \alpha_2 ISD^2_{it} + e_{it} \]

(R2) \[ relVol_{it} = \alpha_0 + \alpha_1 ISD_{it} + \alpha_2 ISD^2_{it} + \alpha_3 \mu_{it} + \alpha_4 \mu^2_{it} + \alpha_5 \sigma_{it} + \alpha_6 \sigma^2_{it} + e_{it} \]
Past literature typically uses equation (R1) without the second-order term. In our investigation we include the second-order terms to capture the higher order effects to explain the annualized realized volatility \( (relVol_{t,T}) \), where \( t \) is the date of observation of option prices for a given stock, and \( T \) is the maturity date. To compute the ISD, we use the dividend adjusted Black-Scholes implied volatilities given in the OptionMetrics data file. \( ISD_{t,T} \) is the average of these implied volatilities of all options that are used to estimate the \( \mu_{t,T} \) and \( \sigma_{t,T} \) pair. Since we take near-the-money options with same days-to-maturity to estimate \( \mu_{t,T} \) and \( \sigma_{t,T} \), there is not much difference among the Black-Scholes implied volatilities of these options. To compute \( relVol_{t,T} \), first, we compute daily realized volatility \( rel\sigma_{Daily} \) based on ex-post daily returns\(^{10}\) of the underlying asset for the remaining life of the option using the following formula:

\[
rel\sigma_{Daily} = \sqrt{\frac{1}{\tau-1} \sum_{i=1}^{\tau} (u_i - u_{i, bar})^2}
\]

where \( \tau \) is the remaining life (in working days) of the option; \( u_i = \ln(1+r_i) \); \( r_i \) is the daily return of the underlying asset for day \( i \) in CRSP database; \( u_{i, bar} \) is the mean of the \( u_i \) series. Then, \( relVol_{t,T} \) is given by \( rel\sigma_{Daily} \sqrt{252} \). Table II and III shows the summary statistics of realized volatilities \( (relVol_{t,T}) \) of all stock options, and S&P500 index options respectively for different groups of day-to-maturity of options.

Using the above regression models of equation (R1) and equation (R2) we can test three hypotheses. First, we can verify if the coefficients of \( \mu \) and \( \sigma \) are significant even in the presence of ISD. Second, we can compare the significance of \( \alpha_0 \) in these two

\(^{10}\) Hull (2002) uses a similar procedure to compute realized volatilities.
specifications. If \( \alpha_0 \) is significant that will imply additional information that are not explained by the independent variables. If \( \alpha_0 \) is insignificant, then the opposite holds true.

Finally, we can test the hypothesis \( H_0: \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0 \). If we reject this null hypothesis then we can argue that \( \mu \) and \( \sigma \) have significant contribution in forecasting the future volatility using the model as given in equation (R2). The results of regression for all stock options are shown in Table V. We have separate regression for different days-to-maturity groups. As before, if days-to-maturity is less than or equal to 30 days then the observation is in 30 days-to-maturity group. If days-to-maturity is greater than 30 but less than or equal to 60 then the observation is in 60 days-to-maturity group and so on. Our estimates of \( \mu \) and \( \sigma \) are based on month-end observation of option prices. Therefore, options with maturity of higher than 30 days will have overlapping observations in the regression. As expected, for the regressions with days-to-maturity group of higher than 30 days, the Durbin-Watson (DW) statistics before AR1 correction (not shown in the table) are significantly different from two. The DW statistics for 30 days-to-maturity are not different from two and therefore we do not need the AR1 correction. The estimates of DW statistics shown in the table are with the correction for days-to-maturity of higher than 30 days, and are without the correction for 30 days-to-maturity group.

How does equation (R1) compare with equation (R2) in explaining the realized volatility?

To address this question first we see for 30 days-to-maturity, coefficients of \( \mu_{tT}^2 \) and \( \sigma_{tT}^2 \) are significant. For 60 days-to-maturity coefficients of \( \mu_{tT} \), \( \sigma_{tT} \), \( \mu_{tT}^2 \), and \( \sigma_{tT}^2 \) are significant. For 90 days-to-maturity (not shown) these four coefficients are also significant. Second, for lower days-to-maturity the coefficients of the intercept term are insignificant in equation (R2). However, these are significant in equation (R1). This shows in equation (R1), even though ISD explains a significant portion of realized volatility, there is a large unexplained intercept in the regression. Whereas equation (R2) provides a better model for
lower-days-to-maturity such that the unknown intercept term is insignificant. For higher
days-to-maturity, the intercept term remains significant for both these equations.
Finally, we use the likelihood ratio to test the hypothesis $H_0: \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0$. The
likelihood ratios are significant in our experiment. Therefore, we reject the restricted model
as given in equation (R1) for all maturity groups shown in Table V. These results indicate,
especially for short-term maturity options, inclusion of $\mu$, $\sigma$, and their second-order terms
provides a better model than simply using Black-Scholes implied volatility to forecast the
realized volatility.

6. Conclusion

This paper uses a risk-adjusted method for joint estimation of implied expected stock return
and volatility from market observed option prices. We find that investors in option markets
have a higher expectation of stock return in the short-term, but a lower expectation of stock
return in the long-term. This finding is robust to different measures of option prices such as
the European price and the American price. This term structure of expected stock return
also remains for deep-in and deep-out of the money call options (not shown here). We also
find that the market friction proxies such as volume, open interest and bid-ask spread do not
explain this term structure. This term structure persists for combined call and put options
(not shown here). This term structure finding supports McNulty et al. (2002) explanation
where the authors argue that shorter horizon investments should be discounted at a higher
rate. However, they use a heuristic approach without a theoretical setting to arrive at these
results. On the other hand, our paper provides the necessary theoretical support for this
finding. We also find that a combination of our implied expected return and implied
volatility with Black-Scholes implied standard deviation (ISD) provides a better model,
than using ISD alone to forecast future volatility of the stock using the short-term options.
There are many possibilities of future research using our findings. Our approach possibly can be used to estimate the cost of equity for investment projects. Especially estimates of expected return for one-year or more will have lower standard error, which is a necessary condition for this to be useful as an estimate of cost of equity. One of the advantages of our approach is that the expected return of a stock can be computed without using any information of the market portfolio such as the market risk premium. Moreover, our results can be deduced without assuming a utility structure for the representative agent. Furthering the research, we plan to investigate whether the term structure persists using other approaches. Nonetheless, better forecasting capability of future volatility using our expected return and volatility might suggest additional investigation of information content of these findings.

References


Appendix

A. Proof of Proposition 1:

We follow Black-Scholes (1973) no-arbitrage argument but we do not assume the CAPM. Let the price change for the stock and option during a small interval of time \( \Delta t \) are \( \Delta S \) and \( \Delta C \) respectively. Without loss of generality, we assume \( t \) as the current time. Let the current stock and option prices are \( S_t \) and \( C_t \) respectively. This implies:

\[
\frac{\Delta S}{S_t} = r_S \\
\frac{\Delta C}{C_t} = r_C
\]

(A1) \[
E[r_S] = \mu \Delta t \\
E[r_C] = k \Delta t
\]

When \( \Delta t \) is a small interval of time, then \( \Delta t \) tends to \( dt \), \( \Delta S \) tends to \( dS \), and \( \Delta C \) tends to \( dC \).

Since stock price \( S \) follows a geometric Brownian, the change in the price of the stock \( \Delta S \) during the small interval of time \( \Delta t \) is:

(A2) \[
ds = \mu S_t dt + \sigma S_t dW
\]

where \( dW \) is the Wiener differential. Then, following Ito’s Lemma, option price change is given by:
\[ \frac{dC}{dS} = \frac{\partial C}{\partial S} dS + \left( \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S_t^2 + \frac{\partial C}{\partial t} \right) dt \]
\[ = \frac{\partial C}{\partial S} dS + \left( rC_t - \frac{\partial C}{\partial S} rS_t \right) dt \]

where the second line of (A3) is derived from the Black-Scholes PDE (partial differential equation). From (A3), we can then compute the covariance between the option return and the stock return as follows:

\[ \text{cov} \left[ \frac{dC}{C_t}, \frac{dS}{S_t} \right] = \frac{1}{C_t S_t} \text{cov}[dC, dS] \]
\[ = \frac{1}{C_t S_t} \frac{\partial C}{\partial S} \text{var}[dS] \]
\[ = \frac{S_t \frac{\partial C}{\partial S} \text{var} \left[ \frac{dS}{S_t} \right]}{C_t \frac{\partial C}{\partial S} \text{var} \left[ \frac{dS}{S_t} \right]} \]

Then it follows that:

\[ \frac{S_t \frac{\partial C}{C_t} \frac{dS}{\partial S}}{\text{var} \left[ \frac{dS}{S_t} \right]} = \beta \]

Finally, taking the expectation of (A3), we obtain:

\[ k dt = \beta \mu dt + r(1 - \beta) dt \]

Q.E.D.
Further, we note that, if we take covariance of both sides of (A3) with respect to the market return $r_M$, then we will obtain the following:

\[(A7) \quad k = r + \beta (\mu - r)\]

where

\[\beta = \frac{\beta_C}{\beta_S}\]

\[\beta_C = \frac{\text{cov}(r_C, r_M)}{\text{var}(r_M)}\]

\[\beta_S = \frac{\text{cov}(r_S, r_M)}{\text{var}(r_M)}\]

This implies:

\[(A8) \quad \beta = \frac{\text{cov}(r_C, r_S)}{\text{var}(r_S)} = \frac{\text{cov}(r_C, r_M)}{\text{cov}(r_S, r_M)}\]

**B. Proof of Proposition 2:**

From (5), we can compute the expected value of the call payoff:

\[(B1) \quad E[C_T] = e^{k(T-t)}C_t\]

\[= S_t e^{\mu(T-t)}N(h_1) - KN(h_2)\]

From the known result of the moment generating function of a Gaussian variable, we have:
\[
\text{var}[S_T] = E[S_T^2] - (E[S_T])^2 \\
= s_t^2 e^{(2\mu + \sigma^2)(T-t)} - s_t^2 e^{2\mu(T-t)} \\
= s_t^2 e^{2\mu(T-t)} \left( e^{\sigma^2(T-t)} - 1 \right)
\]

and

\[
E[S_T^2 C_T] = \int_0^\infty S_T \text{max}\{S_T - K, 0\} \phi(S_T)dS_T \\
= \int_0^K S_T \phi(S_T)dS_T - K \int_K^\infty S_T \phi(S_T)dS_T \\
= s_t^2 e^{(2\mu + \sigma^2)(T-t)} N(h_3) - KS_t e^{\mu(T-t)} N(h_1)
\]

where

\[
h_3 = \frac{\ln S - \ln K + (\mu + \frac{3}{2} \sigma^2)(T - t)}{\sigma \sqrt{T - t}}
\]

Hence, the covariance term in (7a) can be computed as:

\[
= s_t^2 e^{(2\mu + \sigma^2)(T-t)} N(h_3) - KS_t e^{\mu(T-t)} N(h_1) - S_t e^{\mu(T-t)} \left[ S_t e^{\mu(T-t)} N(h_1) - KN(h_2) \right] \\
= s_t^2 e^{2\mu(T-t)} \left[ e^{\sigma^2(T-t)} N(h_3) - \frac{K}{S_t} e^{-\mu(T-t)} \left( N(h_1) - N(h_2) \right) \right] - N(h_1)
\]

Finally, combining equations (7a), (B2), and (B4) we have:
\[
\beta = \frac{S_t \left[ e^{\sigma^2(T-t)} N(h_3) - \frac{K}{S_t} e^{-\mu(T-t)} \left( N(h_1) - N(h_2) \right) - N(h_1) \right]}{C_t \left( e^{\sigma^2(T-t)} - 1 \right)}
\]

Q.E.D.
Table I: Input Data Summary Statistics of All Options

The sample consists of all month-end near-the-money U.S. exchange traded call options for the period of January 1996- April 2006. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days-to-maturity is less than or equal to 30 days then the observation is in 30 days-to-maturity group. If days-to-maturity is between 30 and 60 days then the observation is in 60 days-to-maturity group and so on. Moneyness we define as the stock price divided by the strike price. Avg. volume is the average of volume of call options used for a mu and sigma estimate. Avg. spread is the average of spread of call options used for a mu and sigma estimate. Spread is defined as (offer - bid)/call price. Call price is the mid point of bid and offer or the European option price whichever is lower. European option price is computed from Black-Scholes implied volatility in the data. Number of calls used is the number of option records that are used to compute a mu and sigma pair.

<table>
<thead>
<tr>
<th>Days-to-maturity groups</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>180</th>
<th>360</th>
<th>540</th>
<th>720</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations</td>
<td>11565</td>
<td>8881</td>
<td>3240</td>
<td>2763</td>
<td>4373</td>
<td>3371</td>
<td>642</td>
<td>730</td>
</tr>
<tr>
<td>Variety Mean</td>
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<td>49.99257</td>
<td>80.2608</td>
<td>110.8064</td>
<td>155.5751</td>
<td>239.0389</td>
<td>449.9486</td>
<td>708.0863</td>
</tr>
<tr>
<td>Avg. moneyness Mean</td>
<td>0.998656</td>
<td>0.997899</td>
<td>0.997736</td>
<td>0.998565</td>
<td>0.997883</td>
<td>0.998086</td>
<td>0.997091</td>
<td>0.998257</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.010261</td>
<td>0.010452</td>
<td>0.010799</td>
<td>0.010609</td>
<td>0.0107</td>
<td>0.010787</td>
<td>0.010664</td>
<td>0.010765</td>
</tr>
<tr>
<td>Min</td>
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<td>0.954817</td>
<td>0.958715</td>
<td>0.959115</td>
<td>0.959948</td>
<td>0.962419</td>
<td>0.965038</td>
<td>0.962616</td>
</tr>
<tr>
<td>Max</td>
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<td>1.034318</td>
<td>1.03887</td>
<td>1.034853</td>
<td>1.040099</td>
<td>1.030084</td>
<td>1.032731</td>
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<td>0.003429</td>
<td>-0.06324</td>
<td>-0.02674</td>
<td>-0.01597</td>
<td>-0.00096</td>
<td>0.032379</td>
</tr>
<tr>
<td>Kurt</td>
<td>-0.35879</td>
<td>-0.28402</td>
<td>-0.26315</td>
<td>-0.16634</td>
<td>-0.30006</td>
<td>-0.08729</td>
<td>0.056619</td>
<td>-0.01483</td>
</tr>
<tr>
<td>Median</td>
<td>0.998601</td>
<td>0.997899</td>
<td>0.997665</td>
<td>0.998693</td>
<td>0.997589</td>
<td>0.99825</td>
<td>0.997163</td>
<td>0.998202</td>
</tr>
</tbody>
</table>

| Number of calls used Mean | 2.364462 | 2.312577 | 2.357099 | 2.211726 | 2.1866 | 2.179769 | 2.160436 | 2.156164 |
| Std. Dev.                | 1.638055 | 1.274061 | 1.154322 | 0.754696 | 0.662989 | 0.599665 | 0.570293 | 0.465839 |
| Min                      | 2  | 2  | 2  | 2  | 2  | 2  | 2  | 2  |
| Max                      | 24 | 21 | 14 | 13 | 11 | 10 | 7 | 6 |
| Median                  | 63.9315 | 53.47316 | 29.05353 | 50.64127 | 49.30524 | 37.1084 | 30.67619 | 19.17882 |
| Avg. spread Mean        | 0.135416 | 0.080799 | 0.068408 | 0.059072 | 0.052442 | 0.044398 | 0.03364 | 0.033451 |
| Std. Dev.               | 0.129671 | 0.067326 | 0.060767 | 0.046554 | 0.047561 | 0.030124 | 0.03014 | 0.024066 |
| Min                     | -0.46121 | -0.36306 | -0.20916 | -0.15284 | -1.33358 | -0.10263 | -0.52822 | -0.03165 |
| Max                     | 1.625 | 1.227679 | 1.714286 | 1.153846 | 1.645161 | 0.62079 | 0.209304 | 0.253661 |
| Median                  | 19.64506 | 60.24914 | 224.2863 | 145.7877 | 465.5045 | 59.71149 | 169.6177 | 18.15593 |
| Avg. volume Mean        | 0.102403 | 0.068027 | 0.058277 | 0.052668 | 0.04739 | 0.04028 | 0.032072 | 0.029731 |
| Std. Dev.               | 547.0359 | 266.7459 | 206.621 | 157.6575 | 131.0213 | 139.379 | 121.7841 | 104.5247 |
| Min                     | 1786.683 | 814.9684 | 519.0864 | 433.2877 | 459.6463 | 708.1293 | 369.9922 | 453.1043 |
| Max                     | 53213 | 25245 | 10729.25 | 8100 | 13579.5 | 27849 | 6050 | 10015.33 |
| Median                  | 209.261 | 243.5643 | 88.98739 | 111.7075 | 331.5816 | 804.7501 | 117.5213 | 322.9261 |
| Total open interest Mean| 12396.11 | 7452.418 | 11799.22 | 9480.218 | 7398.776 | 8462.374 | 16587.81 | 8422.127 |
| Std. Dev.               | 42459 | 27743.11 | 29327.43 | 24452.97 | 20893.22 | 25813.37 | 37569.38 | 24622.19 |
| Min                     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Max                     | 984687 | 1003966 | 541457 | 499967 | 514526 | 445689 | 321969 | 293838 |
| Median                  | 145.2858 | 280.2584 | 114.3287 | 119.6839 | 201.096 | 100.5444 | 29.6288 | 53.45322 |

| Total open interest Mean | 3161 | 1344 | 3406.5 | 2698 | 2053 | 1314 | 4715.5 | 1667 |

30
Table II: Implied and Realized Summary Statistics Using All Options

The sample consists of all month-end near-the-money U.S. exchange traded call options for the period of January 1996- April 2006. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days-to-maturity is less than or equal to 30 days then the observation is in 30 days-to-maturity group. If days-to-maturity is greater than 30 but less than or equal to 60 then the observation is in 60 days-to-maturity group and so on. We use all the call options on the same CUSIP, days-to-maturity, and trade date to compute the implied expected stock return and implied volatility by a grid search method that minimizes the square of difference between the observed and computed option price. Realized volatility is computed based on actual return of the stock from trade date to maturity date of the option. Results are shown in decimals.

<table>
<thead>
<tr>
<th>Days-to-maturity groups</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>180</th>
<th>360</th>
<th>540</th>
<th>720</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implied expected return</strong></td>
<td>Mean</td>
<td>0.430065</td>
<td>0.274133</td>
<td>0.21481</td>
<td>0.187903</td>
<td>0.159334</td>
<td>0.129949</td>
<td>0.107797</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.230455</td>
<td>0.131472</td>
<td>0.099885</td>
<td>0.085866</td>
<td>0.075132</td>
<td>0.05942</td>
<td>0.0477</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>2</td>
<td>0.820043</td>
<td>0.582205</td>
<td>0.483588</td>
<td>1.504852</td>
<td>0.374305</td>
<td>0.240838</td>
</tr>
<tr>
<td></td>
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<td>1.001797</td>
<td>0.728487</td>
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<td>0.460138</td>
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<td>0.550249</td>
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<tr>
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<td>0.255299</td>
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<td>0.180111</td>
<td>0.15197</td>
<td>0.125633</td>
<td>0.107531</td>
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<tr>
<td><strong>Implied volatility</strong></td>
<td>Mean</td>
<td>0.443168</td>
<td>0.432945</td>
<td>0.411373</td>
<td>0.416079</td>
<td>0.413623</td>
<td>0.390927</td>
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<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.236102</td>
<td>0.225957</td>
<td>0.216578</td>
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<td>0.206093</td>
<td>0.189718</td>
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<tr>
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<td>Min</td>
<td>0.04648</td>
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</tr>
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</tr>
<tr>
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<td>1.163662</td>
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<td>1.129719</td>
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</tr>
<tr>
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<tr>
<td></td>
<td>Median</td>
<td>0.371658</td>
<td>0.372446</td>
<td>0.35557</td>
<td>0.36157</td>
<td>0.35987</td>
<td>0.342989</td>
<td>0.361167</td>
</tr>
<tr>
<td><strong>Realized volatility</strong></td>
<td>Mean</td>
<td>0.410876</td>
<td>0.402699</td>
<td>0.384182</td>
<td>0.397161</td>
<td>0.402098</td>
<td>0.383391</td>
<td>0.400312</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
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<td>0.299004</td>
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<td>0.285333</td>
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</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.020839</td>
<td>0.031882</td>
<td>0.025105</td>
<td>0.025999</td>
<td>0.012713</td>
<td>0.016992</td>
<td>0.077382</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>3.957029</td>
<td>3.281549</td>
<td>2.15659</td>
<td>2.071244</td>
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<td>2.522718</td>
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<tr>
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<td>Skew</td>
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<tr>
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<td>0.318625</td>
<td>0.310317</td>
<td>0.337483</td>
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</tbody>
</table>
### Table III: Implied and Realized Summary Statistics Using S&P500 Index Options

The sample consists of all month-end near-the-money S&P 500 index call options for the period of January 1996- April 2006. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days-to-maturity is less than or equal to 30 days then the observation is in 30 days-to-maturity group. If days-to-maturity is greater than 30 but less than or equal to 60 then the observation is in 60 days-to-maturity group and so on. We use all the call options on the same CUSIP, days-to-maturity, and trade date to compute the implied expected stock return and implied volatility by a grid search method that minimizes the square of difference between the observed and computed option price. Realized volatility is computed based on actual return of the stock from trade date to maturity date of the option. Results are shown in decimals.

<table>
<thead>
<tr>
<th>Days-to-maturity groups</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>180</th>
<th>360</th>
<th>540</th>
<th>720</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implied expected return</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.270438</td>
<td>0.200704</td>
<td>0.166686</td>
<td>0.153305</td>
<td>0.135502</td>
<td>0.108522</td>
<td>0.085861</td>
<td>0.063683</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.110916</td>
<td>0.070695</td>
<td>0.054855</td>
<td>0.048066</td>
<td>0.042791</td>
<td>0.040906</td>
<td>0.036499</td>
<td>0.021914</td>
</tr>
<tr>
<td>Min</td>
<td>0.108889</td>
<td>0.096642</td>
<td>0.082551</td>
<td>0.081111</td>
<td>0.071111</td>
<td>0.024111</td>
<td>0.018889</td>
<td>0.029111</td>
</tr>
<tr>
<td>Max</td>
<td>0.757185</td>
<td>0.431089</td>
<td>0.337511</td>
<td>0.261783</td>
<td>0.235831</td>
<td>0.260083</td>
<td>0.146482</td>
<td>0.104426</td>
</tr>
<tr>
<td>Skew</td>
<td>1.285956</td>
<td>0.575425</td>
<td>0.458549</td>
<td>0.301559</td>
<td>0.322322</td>
<td>0.821879</td>
<td>0.058801</td>
<td>0.706789</td>
</tr>
<tr>
<td>Kurt</td>
<td>2.893653</td>
<td>0.207557</td>
<td>-0.36527</td>
<td>-0.96245</td>
<td>-0.90928</td>
<td>0.534962</td>
<td>-0.92896</td>
<td>0.309432</td>
</tr>
<tr>
<td>Median</td>
<td>0.255158</td>
<td>0.200566</td>
<td>0.16002</td>
<td>0.14594</td>
<td>0.12966</td>
<td>0.093804</td>
<td>0.086443</td>
<td>0.056673</td>
</tr>
<tr>
<td><strong>Implied volatility</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Mean</td>
<td>0.208969</td>
<td>0.227357</td>
<td>0.22732</td>
<td>0.235487</td>
<td>0.235922</td>
<td>0.230475</td>
<td>0.230553</td>
<td>0.192068</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.081675</td>
<td>0.082334</td>
<td>0.078263</td>
<td>0.077275</td>
<td>0.076733</td>
<td>0.100232</td>
<td>0.09084</td>
<td>0.060479</td>
</tr>
<tr>
<td>Min</td>
<td>0.078772</td>
<td>0.103501</td>
<td>0.103448</td>
<td>0.116111</td>
<td>0.092077</td>
<td>0.107189</td>
<td>0.101664</td>
<td>0.117795</td>
</tr>
<tr>
<td>Max</td>
<td>0.541435</td>
<td>0.493003</td>
<td>0.481111</td>
<td>0.442989</td>
<td>0.470111</td>
<td>0.929111</td>
<td>0.431111</td>
<td>0.291111</td>
</tr>
<tr>
<td>Skew</td>
<td>1.097853</td>
<td>0.684949</td>
<td>0.574595</td>
<td>0.344706</td>
<td>0.389002</td>
<td>3.100203</td>
<td>0.560332</td>
<td>0.398041</td>
</tr>
<tr>
<td>Kurt</td>
<td>1.890491</td>
<td>0.495528</td>
<td>0.058466</td>
<td>-0.29823</td>
<td>-0.19136</td>
<td>19.34735</td>
<td>-0.51904</td>
<td>-1.28284</td>
</tr>
<tr>
<td>Median</td>
<td>0.198942</td>
<td>0.220875</td>
<td>0.224429</td>
<td>0.233766</td>
<td>0.238889</td>
<td>0.218895</td>
<td>0.221911</td>
<td>0.180111</td>
</tr>
<tr>
<td><strong>Realized volatility</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.163257</td>
<td>0.169432</td>
<td>0.167046</td>
<td>0.171822</td>
<td>0.169409</td>
<td>0.167617</td>
<td>0.165848</td>
<td>0.134543</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.075872</td>
<td>0.068422</td>
<td>0.065898</td>
<td>0.060734</td>
<td>0.058129</td>
<td>0.060383</td>
<td>0.054013</td>
<td>0.039326</td>
</tr>
<tr>
<td>Min</td>
<td>0.063197</td>
<td>0.074925</td>
<td>0.082902</td>
<td>0.087075</td>
<td>0.088153</td>
<td>0.096029</td>
<td>0.099312</td>
<td>0.098902</td>
</tr>
<tr>
<td>Max</td>
<td>0.43242</td>
<td>0.411919</td>
<td>0.357542</td>
<td>0.325498</td>
<td>0.323015</td>
<td>0.309768</td>
<td>0.251611</td>
<td>0.200148</td>
</tr>
<tr>
<td>Skew</td>
<td>1.288415</td>
<td>1.147816</td>
<td>1.040592</td>
<td>0.549189</td>
<td>0.525578</td>
<td>0.385895</td>
<td>-0.04382</td>
<td>0.826683</td>
</tr>
<tr>
<td>Kurt</td>
<td>1.479818</td>
<td>1.172168</td>
<td>0.517725</td>
<td>-0.41888</td>
<td>-0.47728</td>
<td>-1.08762</td>
<td>-1.62703</td>
<td>-1.22102</td>
</tr>
<tr>
<td>Median</td>
<td>0.142581</td>
<td>0.157326</td>
<td>0.154478</td>
<td>0.168416</td>
<td>0.170843</td>
<td>0.176483</td>
<td>0.183594</td>
<td>0.114379</td>
</tr>
</tbody>
</table>
Table IV: Results from Regressing Mu on the Indicated Variables

This table presents results from regression of mu (μ) levels for different days-to-maturity groups using S&P500 index option data. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days-to-maturity is less than or equal to 30 days then the observation is in 30 days-to-maturity group. If days-to-maturity is greater than 30 but less than or equal to 60 then the observation is in 60 days-to-maturity group and so on. The values in parenthesis are the t-statistics. AvgMoneyness is average of the stock price divided by the strike price of options used to compute mu. AbsRet, LAbsRet, L2AbsRet are the current and first two lagged daily absolute returns of the S&P 500 index. AvgSpread is average of (offer-bid)/call price of all option records used to compute mu. TotalOpnInt is the total option interest of the options used to compute mu. AvgVolume is the average volume, and RecCount is the number of records used to compute mu. Ret, LRet, L2Ret are the current and first two lagged daily returns of the S&P 500 index. DW is the Durbin-Watson statistics. **, and * represents the p-values of less than 0.01, and between 0.01 and 0.05 respectively.

<table>
<thead>
<tr>
<th>Days-to-maturity groups</th>
<th>30 Days</th>
<th>60 Days</th>
<th>180 Days</th>
<th>360 Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-1.8473 (-1.51)</td>
<td>-1.7077** (-4.14)</td>
<td>-0.2631 (-1.23)</td>
<td>-0.3905** (-3.61)</td>
</tr>
<tr>
<td>AvgMoneyness</td>
<td>2.3082 (1.88)</td>
<td>1.8897** (4.59)</td>
<td>0.4556* (2.16)</td>
<td>0.5045** (4.67)</td>
</tr>
<tr>
<td>DaysToMaturity</td>
<td>-0.0070** (-3.53)</td>
<td>0.0007 (0.64)</td>
<td>-0.0003* (-2.59)</td>
<td>-8.6E-05** (-3.07)</td>
</tr>
<tr>
<td>AbsRet</td>
<td>3.2930** (3.47)</td>
<td>1.3033** (3.39)</td>
<td>0.6096 (1.73)</td>
<td>0.3467 (1.2)</td>
</tr>
<tr>
<td>L.AbsRet</td>
<td>0.9826 (1.12)</td>
<td>0.6551 (1.93)</td>
<td>0.3689 (1.25)</td>
<td>-0.0438 (-0.18)</td>
</tr>
<tr>
<td>L2.AbsRet</td>
<td>1.2872 (1.56)</td>
<td>0.4005 (1.23)</td>
<td>0.3194 (1.19)</td>
<td>-0.6744** (-2.81)</td>
</tr>
<tr>
<td>AvgSpread</td>
<td>-0.1468* (-2.16)</td>
<td>-0.0827 (-0.59)</td>
<td>-0.0150 (-0.07)</td>
<td>0.7868** (6.82)</td>
</tr>
<tr>
<td>TotalOpnInt</td>
<td>-4.4E-08 (-0.72)</td>
<td>4.22E-08 (1.04)</td>
<td>-2.8E-08 (-0.33)</td>
<td>-7.9E-08 (-1.02)</td>
</tr>
<tr>
<td>AvgVolume</td>
<td>-5.8E-06 (-0.59)</td>
<td>-3.4E-06 (-0.52)</td>
<td>3.09E-07 (0.04)</td>
<td>-2.3E-06 (-1.28)</td>
</tr>
<tr>
<td>RecCount</td>
<td>-0.0043* (-2.13)</td>
<td>-0.0030* (-2.29)</td>
<td>-0.0039 (-1.65)</td>
<td>0.0051* (2.27)</td>
</tr>
<tr>
<td>Ret</td>
<td>-0.9170 (-1.43)</td>
<td>-0.0683 (-0.25)</td>
<td>-0.3118 (-1.36)</td>
<td>-0.0835 (-0.44)</td>
</tr>
<tr>
<td>LRet</td>
<td>-0.6807 (-1.27)</td>
<td>-0.7662** (-3.44)</td>
<td>0.0127 (0.07)</td>
<td>0.0001 (0)</td>
</tr>
<tr>
<td>L2Ret</td>
<td>-0.6463 (-1.22)</td>
<td>-0.2560 (-1.21)</td>
<td>0.1425 (0.77)</td>
<td>-0.1507 (-0.98)</td>
</tr>
<tr>
<td>AR1</td>
<td>-0.4963** (-5.76)</td>
<td>-0.8277** (-14.94)</td>
<td>-0.8597** (-13.23)</td>
<td>-0.8615** (-17.36)</td>
</tr>
<tr>
<td>adjR2</td>
<td>0.4636</td>
<td>0.366</td>
<td>0.3214</td>
<td>0.5091</td>
</tr>
<tr>
<td>DW</td>
<td>2.0798</td>
<td>2.1669</td>
<td>2.0898</td>
<td>2.4359</td>
</tr>
</tbody>
</table>
This table presents the regression results of realized volatility on mu (μ), sigma (σ), and B-S implied volatility for different days-to-maturity groups for the period of January 1996- April 2006 using all stock mu, and sigma. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days-to-maturity is less than or equal to 30 days then the observation is in 30 days-to-maturity group. If days-to-maturity is greater than 30 but less than or equal to 60 then the observation is in 60 days-to-maturity group and so on. Values in parenthesis show the t-statistics. Dependent variable is realized volatility of the stock for the period of the call option. B-S Impl. Vol. is B-S implied volatility, B-S Impl. Vol. Sq. is square of B-S implied volatility, Mu and Sigma are the estimated values, Mu Sq. is square of mu, and Sigma Sq. is square of sigma. LR is the likelihood ratio to test whether the restricted regressions are valid. Restricted regressions do not have the independent variables Mu, Mu Sq., Sigma, and Sigma Sq. in the equation. Serial correlation correction is made for all groups except for 30 days-to-maturity. AR1 shows the serial correlation correction. DW shows the Durbin-Watson without correction for 30 days-to-maturity, and after correction for all other groups. **, and * represents the p-values of less than 0.01, and between 0.01 and 0.05 respectively.

<table>
<thead>
<tr>
<th>Intercept</th>
<th>B-S Impl. Vol.</th>
<th>BS- Impl. Vol Sq.</th>
<th>Mu</th>
<th>Mu Sq.</th>
<th>Sigma</th>
<th>Sigma Sq.</th>
<th>AR1</th>
<th>Adj. R²</th>
<th>DW</th>
<th>LR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>30 Days-to-maturity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.0387**(-6.12)</td>
<td>1.1119**(44.28)</td>
<td>-0.001112(-0.06)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.6838</td>
<td>2.0131</td>
<td>75.22**</td>
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<tr>
<td>-0.00586(-0.67)</td>
<td>1.1534**(9.86)</td>
<td>-0.1183*(-2.29)</td>
<td>0.0476(1.28)</td>
<td>-0.0923*(-3.16)</td>
<td>-0.233(-1.86)</td>
<td>0.3233**(5.08)</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>60 Days-to-maturity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.1247**(-11.48)</td>
<td>0.7496</td>
<td>156.33**</td>
<td></td>
</tr>
<tr>
<td>-0.0321*(5.1)</td>
<td>1.1008**(41.66)</td>
<td>0.0708*(3.19)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.647**</td>
<td>1.9943</td>
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<td></td>
</tr>
<tr>
<td>0.005617(0.66)</td>
<td>1.5148**(13.88)</td>
<td>-0.3337*(-6.35)</td>
<td>0.1605**(2.75)</td>
<td>-0.3724*(-4.32)</td>
<td>-0.6422*(-5.78)</td>
<td>0.635**(10.35)</td>
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</tr>
<tr>
<td><strong>180 Days-to-maturity</strong></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td>-0.1273**(-11.74)</td>
<td>0.7532</td>
<td>19943</td>
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</tr>
<tr>
<td>-0.1242**(-16.73)</td>
<td>1.6102**(55.42)</td>
<td>-0.3295*(-13.97)</td>
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<td>0.777**</td>
<td>1.9875</td>
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</tr>
<tr>
<td>-0.0218*(-1.95)</td>
<td>2.1124**(16.65)</td>
<td>-0.6767*(-10.48)</td>
<td>-0.0223(-0.25)</td>
<td>0.1363(0.95)</td>
<td>-0.9369*(-7.81)</td>
<td>0.7429**(12.69)</td>
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<tr>
<td><strong>360 Days-to-maturity</strong></td>
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<td></td>
<td>-0.3099**(-20.47)</td>
<td>0.7479</td>
<td>236.31**</td>
<td></td>
</tr>
<tr>
<td>-0.0665**(6.22)</td>
<td>1.3456**(27.52)</td>
<td>-0.0706(-1.53)</td>
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<td></td>
<td>0.7384</td>
<td>2.0389</td>
<td>58.31**</td>
<td></td>
</tr>
<tr>
<td>-0.0756**(-5.89)</td>
<td>2.2302**(16.83)</td>
<td>-0.6618*(-6.57)</td>
<td>0.4205**(3.18)</td>
<td>-0.9503(-1.92)</td>
<td>-0.9256*(-7.58)</td>
<td>0.6172**(6.35)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>All Days-to-maturity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.3565**(-21.46)</td>
<td>0.7458</td>
<td>20352</td>
<td></td>
</tr>
<tr>
<td>-0.0643**(19.48)</td>
<td>1.2776**(93.87)</td>
<td>-0.0906*(-8.06)</td>
<td></td>
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<td>0.7047</td>
<td>2.0079</td>
<td>96.244**</td>
<td></td>
</tr>
<tr>
<td>-0.0124**(-2.93)</td>
<td>1.4402**(33.96)</td>
<td>-0.3146*(-14.47)</td>
<td>-0.1023*(-7.22)</td>
<td>-0.0296*(-2.08)</td>
<td>-0.31*(-7.75)</td>
<td>0.4043**(16.62)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11 Mu is option implied expected return, and Sigma is option implied volatility. Mu and Sigma are jointly estimated using options data.
Figure I: Term Structures of Equally-Weighted Average of Mu, Sigma, and B-S Implied Volatility of All Stocks.\(^{12}\)

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\(^{12}\) Mu is option implied expected return, and Sigma is option implied volatility. Mu and Sigma are jointly estimated using options data.
Figure II: Term structures of Equally-Weighted Average of Mu, Sigma, and B-S Implied Volatility of S&P500 Index.\textsuperscript{13}

\textsuperscript{13} Mu is option implied expected return, and Sigma is option implied volatility. Mu and Sigma are jointly estimated using options data.
Mu is option implied expected return, and Sigma is option implied volatility. Mu and Sigma are jointly estimated using options data.
Figure IV: 30 Days Predictability of Realized Sigma by Mu, Sigma, and B-S Implied Volatility of S&P500 Index.\textsuperscript{15}

\textsuperscript{15} Mu is option implied expected return, and Sigma is option implied volatility. Mu and Sigma are jointly estimated using options data.
Figure V: 180 Days Predictability of Realized Sigma by Mu, Sigma, and B-S Implied Volatility of S&P500 Index.\textsuperscript{16}

\textsuperscript{16} Mu is option implied expected return, and Sigma is option implied volatility. Mu and Sigma are jointly estimated using options data.